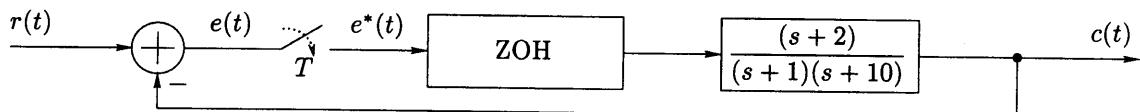


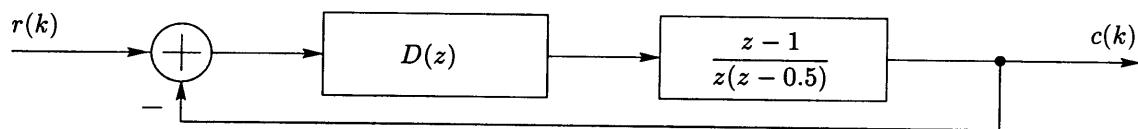
Copyright © 1998 by Levent Acar. All rights reserved. No parts of this document may be reproduced, stored in a retrieval system, or transmitted in any form or by any means without the written permission of the copyright holder(s).

1. Consider the following system with a sampling period of 0.1 second.



Determine the transfer function  $C(z)/R(z)$ . Simplify the result as much as possible. (20pts)

2. Consider the following feedback control system.



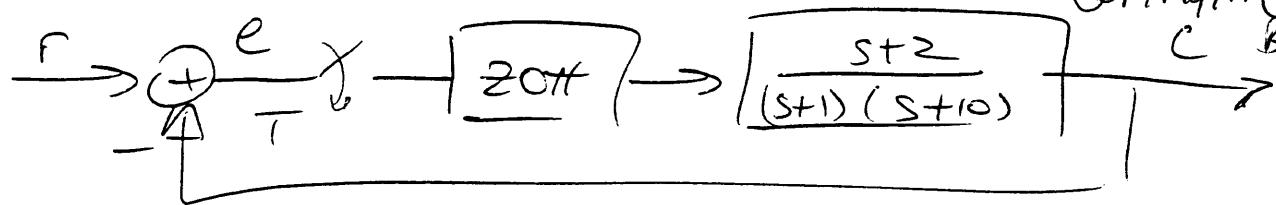
- (a) Assume a proportional controller, i.e.  $D(z) = K$ . Determine the range of stability for all  $K$ . (20pts)
- (b) Design a first-order compensator, such that the closed-loop poles are at  $0.2 \pm j0.2$ . (20pts)

3. The open-loop gain of a unity feedback discrete-time system with a sampling period of 1 second is given by

$$G(z) = \frac{5000(z+1)^3}{2(z-1)(2(z-1)+(z+1))(2(z-1)+1000(z+1))}.$$

Note that the gain  $G(z)$  is given in a special form to simplify the computations.

- (a) Sketch the Bode plots for the system, and determine whether or not the closed-loop system is stable from the plots. (20pts)
- (b) Design either a lead or a lag compensator (not both) to increase the phase margin by another  $20^\circ$ , while keeping the steady state error characteristics the same. (20pts)



$$T = 0.1$$

$$G_{OPEN}(z) = (1 - z^{-1}) \frac{2}{z} \left[ \frac{s+2}{s(s+1)(s+10)} \right]$$

$$= \frac{z-1}{z} \frac{2}{z} \left[ \frac{a}{s} + \frac{b}{s+1} + \frac{c}{s+10} \right]$$

$$a = \left[ \frac{s+2}{(s+1)(s+10)} \right]_{s=0} = \frac{2}{10} = \frac{1}{5}$$

$$b = \left[ \frac{s+2}{s(s+10)} \right]_{s=-1} = \frac{1}{(-1) \times (9)} = -\frac{1}{9}$$

$$c = \left[ \frac{s+2}{s(s+1)} \right]_{s=-10} = \frac{-8}{(-10) \times (-9)} = -\frac{4}{45}$$

$$G_{OPEN}(z) = \frac{z-1}{z} \frac{2}{z} \left[ \frac{1/5}{s} - \frac{1/9}{s+1} - \frac{4/45}{s+10} \right]$$

$$= \frac{z-1}{z} \left[ \frac{1}{5} \frac{2}{z-1} - \frac{1}{9} \frac{2}{z-e^{-0.1}} - \frac{4}{45} \frac{2}{z-e^{-1}} \right]$$

$$= \frac{1}{5} - \frac{1}{9} \frac{2-1}{z-e^{-0.1}} - \frac{4}{45} \frac{2-1}{z-e^{-1}}$$

$$\begin{aligned}
 &= \frac{9(z-e^{-0.1})(z-e^{-1}) - 5(z-1)(z-e^{-1}) - 4(z-1)(z-e^{0.1})}{45(z-e^{-0.1})(z-e^{-1})} \\
 &= \frac{(-4e^{-1} - 5e^{-0.1} + 9)z + (9e^{-1.1} - 5e^{-1} - 4e^{-0.1})}{45(z-e^{-0.1})(z-e^{-1})} \\
 &= \frac{3z - 2.46}{45(z-e^{-0.1})(z-e^{-1})} \\
 &= \frac{0.0667(z-0.8210)}{(z-0.9048)(z-0.3679)}
 \end{aligned}$$

$$\frac{C(z)}{R(z)} = \frac{g_{open}(z)}{1 + g_{open}(z)}$$

$$= \frac{0.0667(z-0.8210)}{(z-0.9048)(z-0.3679) + 0.0667(z-0.8210)}$$

$$= \frac{0.0667(z-0.8210)}{z^2 - 1.206z + 0.2781}$$

$$= \frac{3z - 2.46}{45z^2 - 54.27z + 12.51}$$

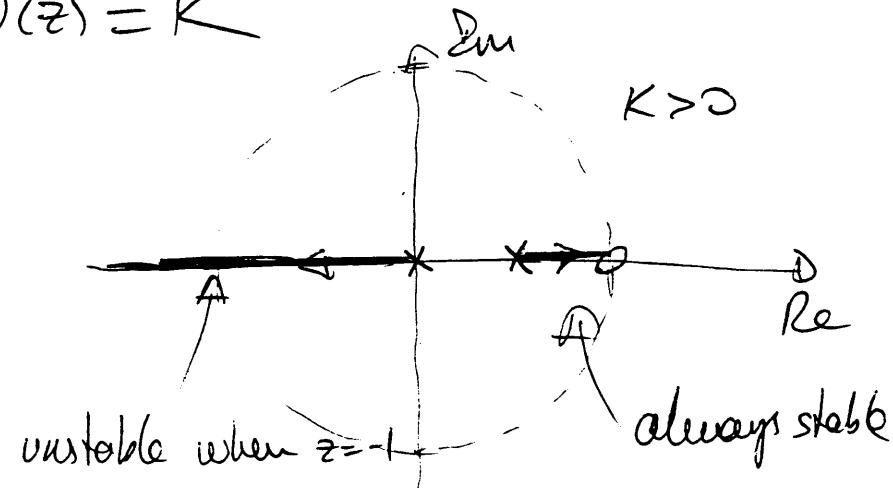
13782 500 SHEETS FILLER 5 SQUARE  
 42391 50 SHEETS EYELET 5 SQUARE  
 42392 100 SHEETS EYELET 5 SQUARE  
 42393 100 RECYCLED WHITE 5 SQUARE  
 42398 200 RECYCLED WHITE 5 SQUARE  
 Made in U.S.A.



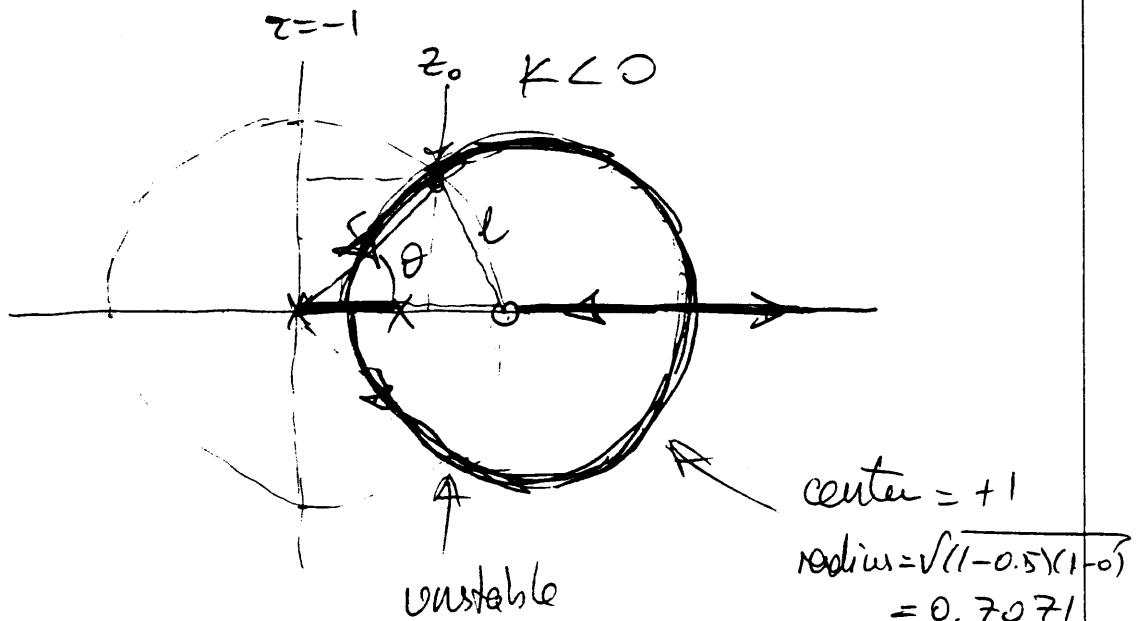
#2

$$\frac{E}{-} \xrightarrow{+} D(z) \xrightarrow{\quad} \left| \frac{z-1}{z(z-0.5)} \right| \xrightarrow{\quad} C$$

(a)  $D(z) = K$



$$\left| K \frac{z-1}{z(z-0.5)} \right| = 1 \Rightarrow K = 0.75$$



$$\begin{aligned} \text{center} &= +1 \\ \text{radius} &= \sqrt{(1-0.5)(1-0)} \\ &= 0.7071 \end{aligned}$$

$$l^2 = r^2 + r^2 - 2r \cdot r \cos \theta \Rightarrow \theta = 41.41^\circ = l$$

$$z_0 = \cos \theta + j \sin \theta = 0.75 + j 0.66$$

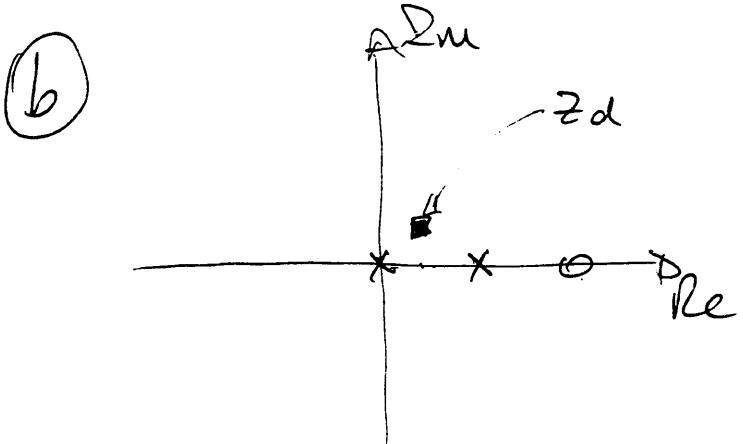
$$\left| K \frac{z-1}{z(z-0.5)} \right| = 1 \Rightarrow K = -1$$

$z = 0.75 + j0.66$

So stable when  $0 \leq K \leq 0.75$   
 $-1 \leq K \leq 0$

or

$$-1 \leq K \leq 0.75$$



let  $D(z) = K \frac{z+a}{z-1}$

$$\Rightarrow D(z) \rho(z) = K \frac{z+a}{z(z-0.5)}$$

Root locus is a circle with center at  $-a$  and radius

$$\sqrt{(0 - (-a))(0.5 - (-a))} = \sqrt{a(0.5 + a)}$$

Equation of such a circle

$$(r+a)^2 + \omega^2 = \theta(0.5+\alpha)$$

To make it pass thru  $z_0 = 0.2 \pm j0.2$

$$\begin{matrix} r \\ \omega \end{matrix}$$

Substitute the variables in

$$(0.2+\alpha)^2 + 0.2^2 = \theta(0.5+\alpha)$$

$$\alpha^2 + 0.4\alpha + 0.04 + 0.04 = \theta^2 + 0.5\alpha$$

$$0.08 = +0.1\alpha$$

$$\alpha = 0.8 \leftarrow$$

This can also be determined by the angular contribution

then  $D(z) = K \frac{z+0.8}{z-1}$

and  $|K \frac{(z+0.8)}{(z-1)} \frac{(z-1)}{z(z-0.5)}| = 1$   
 $\quad \quad \quad z=0.2+j0.2$

$$\Rightarrow K = 0.1$$

or  $D(z) = 0.1 \frac{z+0.8}{z-1}$

EE331

EXHIBIT #1  
80kHz CDS

PART 98 %

#3

 $T=1$ 

$$G(z) = \frac{5000 (z+1)^3}{2(z-1)(2(z-1)+(z+1))(2(z-1)+1000(z+1))}$$

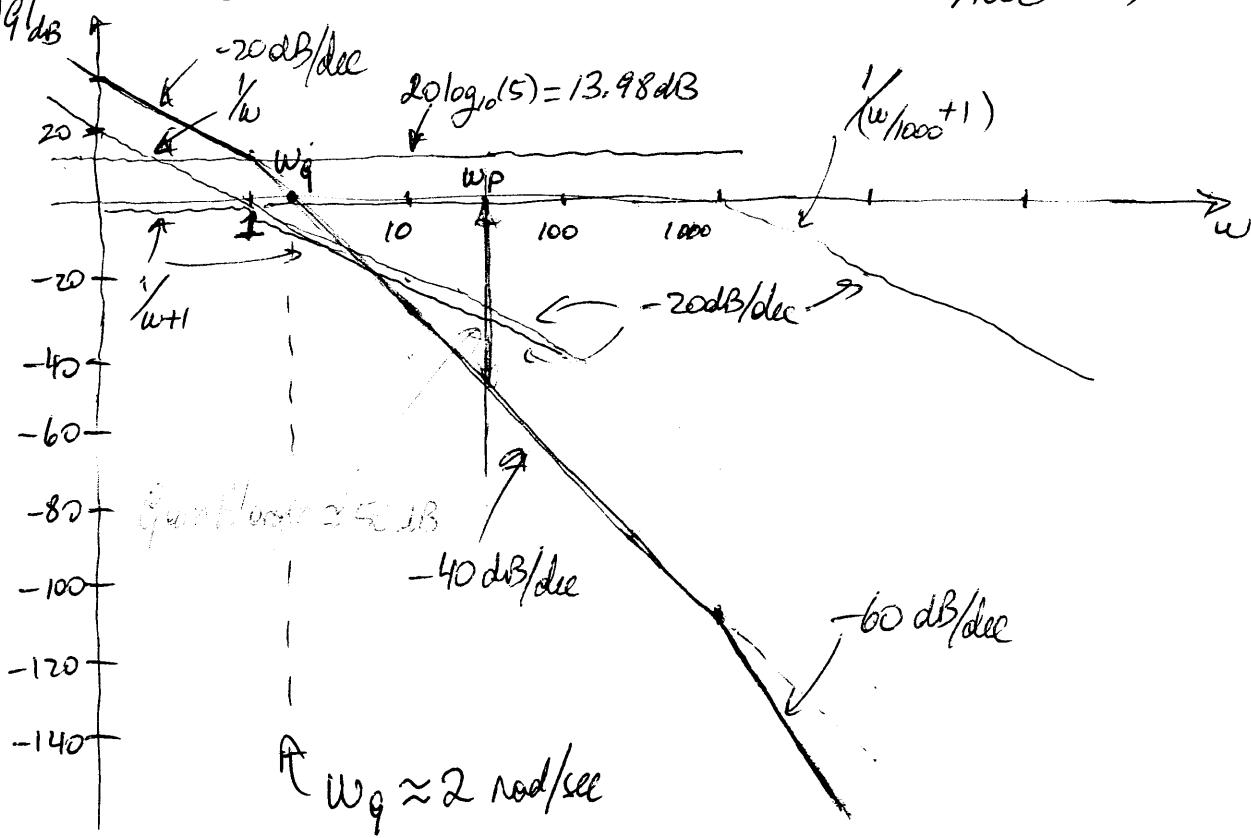
a) For the Bode plot, we need to obtain the  $w$ -domain representation, where

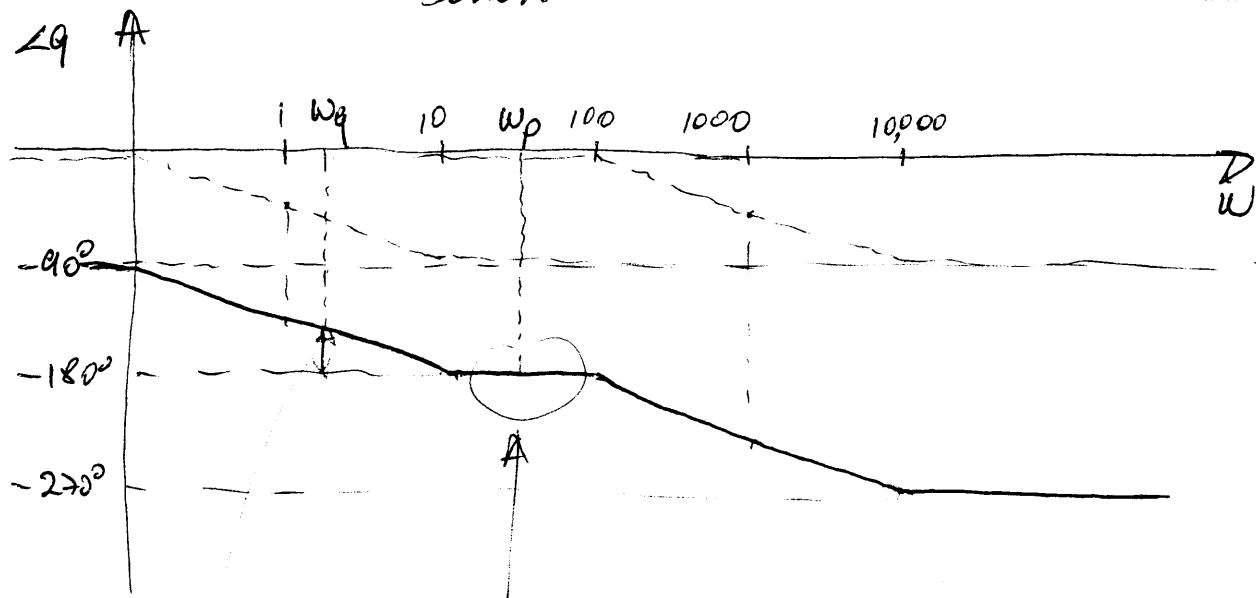
$$z = \frac{1 + (T/2)\omega}{1 - (T/2)\omega} \quad \text{or} \quad \omega = \frac{2}{T} \frac{z-1}{z+1}$$

$$G(w) = \frac{5000}{2 \frac{(z-1)}{(z+1)} \left( 2 \frac{(z-1)}{(z+1)} + 1 \right) \left( 2 \frac{(z-1)}{(z+1)} + 1000 \right)}$$

or

$$G(w) = \frac{5000}{w(w+1)(w+1000)} = \frac{5}{w(w+1)(w/1000+1)}$$





Phase Margin  $\approx 25^\circ$

phase cross-over Frequency  
some where here

$$w_p \approx 40 \text{ rad/sec}$$

### b<sub>11</sub> Lead Compensator

Desired phase angle needed to be added

$$\phi_m = 20^\circ + 5^\circ = 25^\circ$$

The attenuation factor

$$\alpha = \frac{1 - \sin(\phi_m)}{1 + \sin(\phi_m)} = 0.4$$

New gain crossover freq. when

$$|G|_{\text{dB}} = -20 \log \left( \frac{1}{\sqrt{\alpha}} \right)$$

$$= -3.9 \text{ dB}$$

$$\Rightarrow w_m \approx 2.2 \text{ rad/sec}$$

From the plot  $\approx$  very close to  $2 \text{ rad/sec}$

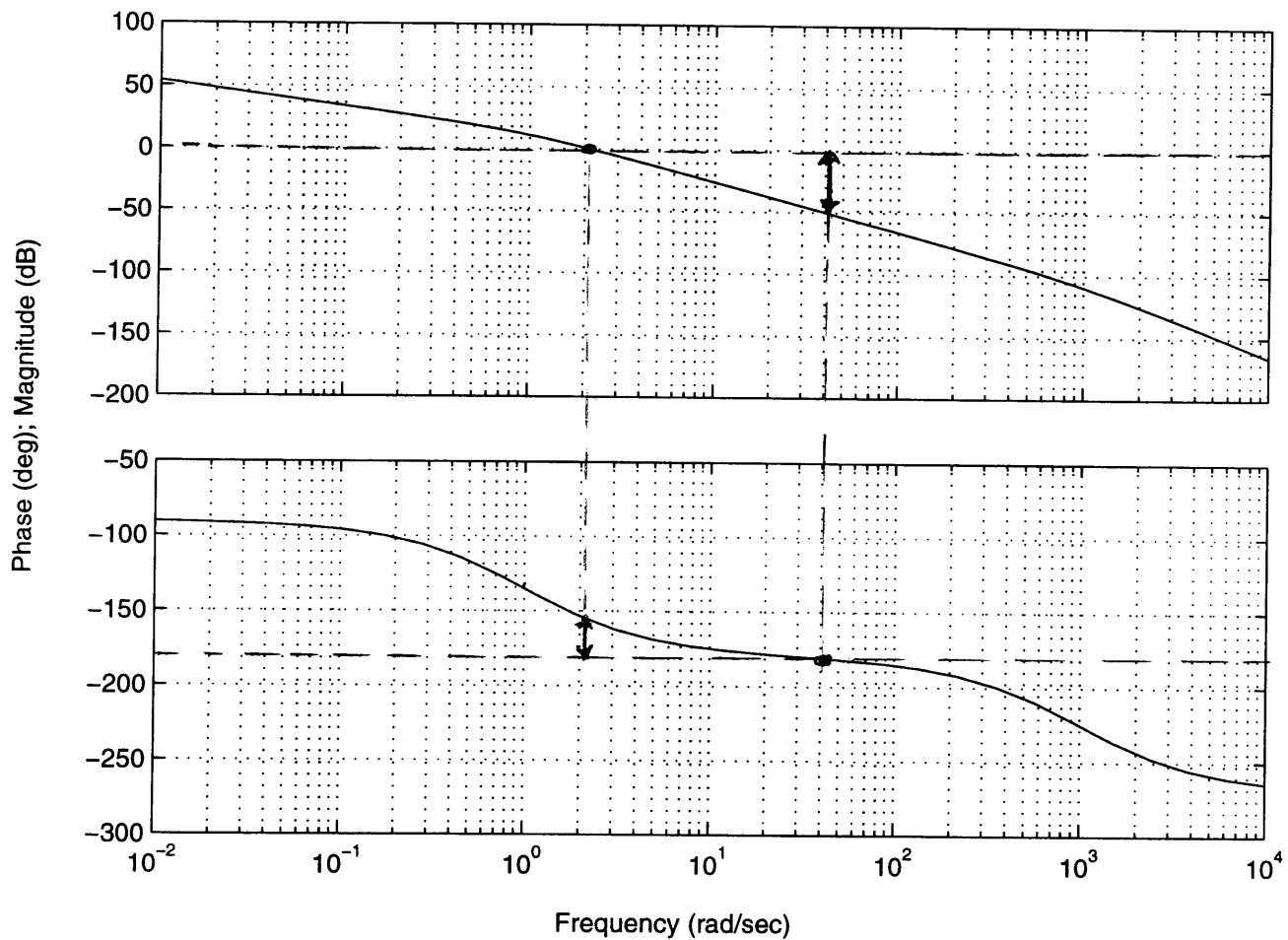
EE331

EXAM #1  
SOLUTIONS

FALL 98

8/10

Bode Diagrams



Corner Frequencies

$$\omega_L = \sqrt{\alpha} \omega_m = 1.39 \text{ rad/sec}$$

$$\omega_{L/\alpha} = 3.48 \text{ rad/sec}$$

$$\Rightarrow D(w) = \frac{\omega/\omega_L + 1}{\omega/\omega_{L/\alpha} + 1} = \frac{\omega/1.39 + 1}{\omega/3.48 + 1}$$

$$= 2.5 \frac{\omega + 1.39}{\omega + 3.48}$$

$$D(z) = D(w) \Big|_{w=2 \frac{(z-1)}{(z+1)}}$$

$$= 2.5 \frac{2(z-1) + 1.39(z+1)}{2(z-1) + 3.48(z+1)}$$

$$D(z) = 1.55 \frac{(z-0.18)}{(z+0.27)}$$

### Lag Compensator

The new gain crossover frequency when

Please margin ↓

$$\angle G = -180^\circ + (25^\circ + 20^\circ) + 5^\circ$$

$$= -135^\circ$$

$$\omega_g \approx 1 \text{ rad/sec} \quad (\text{at } \omega_g, \angle G = -135^\circ)$$

The freq. of  
the zero

$$\omega_L = \omega_Q/10 = 0.1 \text{ rad/sec}$$

The gain at  
 $\omega_Q = 1 \text{ rad/sec}$

$$|G|_{dB} \approx 15 \text{ dB}$$

The attenuation  
coefficient from

$$\Delta G = -|G|_{dB} = -20 \log (\beta)$$

$$-15 = -20 \log (\beta)$$

$$\Rightarrow \beta = 5.6$$

$$\Rightarrow D(w) = \frac{\frac{w}{\omega_L} + 1}{\frac{w}{\omega_Q/\beta} + 1} = \frac{\frac{w}{0.1} + 1}{\frac{w}{0.1/5.6} + 1}$$

$$= 0.18 \frac{w + 0.1}{w + 0.018}$$

$$D(z) = D(w) /$$

$$w = 2 \frac{(z-1)}{(z+1)}$$

$$= 0.18 \frac{2(z-1) + 0.1(z+1)}{2(z-1) + 0.018(z+1)}$$

$$D(z) = 0.19 \frac{z - 0.90}{z - 0.98}$$