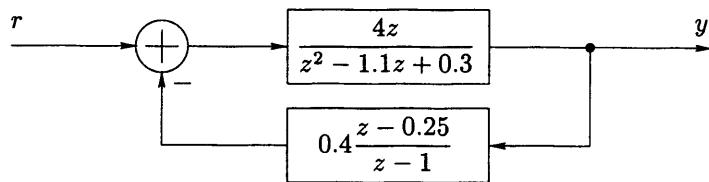


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1. The block diagram of a control system is given in the following figure. Obtain a state-space representation of the system without any block-diagram reduction. (25pts)



2. A continuous-time control system is described by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 10 & 11 \\ -20 & -21 \end{bmatrix} \mathbf{x}(t),$$

where \mathbf{x} is the state vector. Determine its discrete-time state space representation, when $T = 0.5$ second. (25pts)

3. A discrete-time linear control system is described by

$$\begin{aligned} \mathbf{x}(k+1) &= \begin{bmatrix} 1.00 & 2.00 \\ -0.25 & -0.50 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \mathbf{u}(k), \\ \mathbf{y}(k) &= \begin{bmatrix} -1 & 2 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 & 5 \end{bmatrix} \mathbf{u}(k). \end{aligned}$$

Determine whether or not the state $[4 \ -4]^T$ can be achieved from the initial state $[-2 \ 2]^T$ by selecting the control input $\mathbf{u}(k) = [u_1(k) \ u_2(k)]^T$. If such a selection is possible, then determine the control sequence. (25pts)

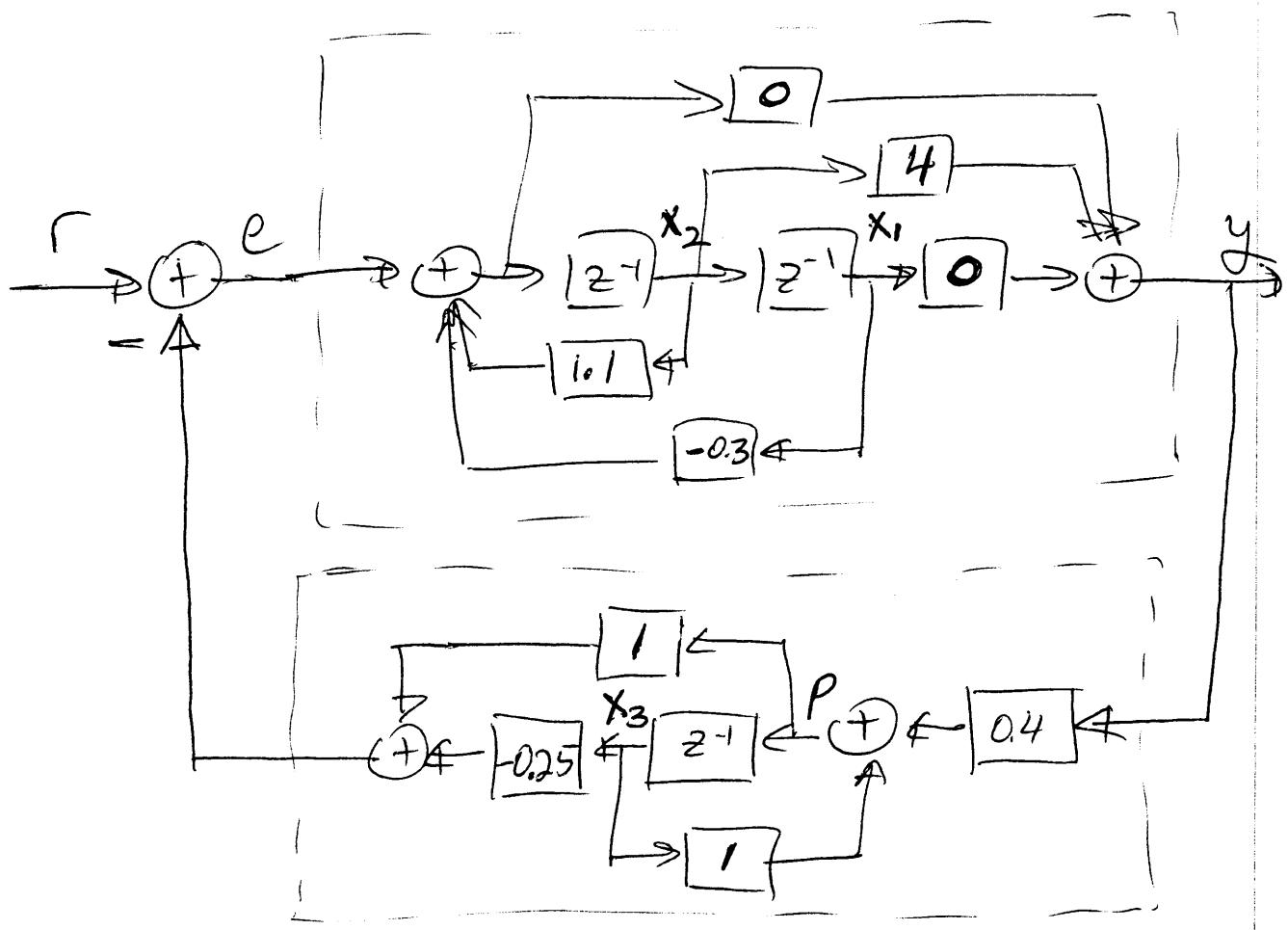
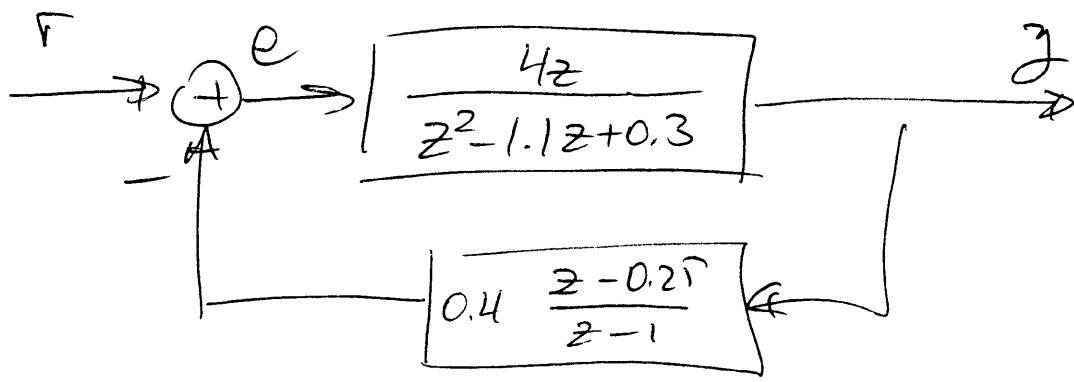
4. A discrete-time linear control system is described by

$$\begin{aligned} \mathbf{x}(k+1) &= \begin{bmatrix} 0.8 & 0.8 \\ 1.1 & 0.4 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{u}(k), \\ \mathbf{y}(k) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 & 1 \end{bmatrix} \mathbf{u}(k). \end{aligned}$$

Design a state-feedback controller, such that the closed-loop poles are at 0.1 and 0.8. (25pts)

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#1



↗
controller canonical form

$$\text{So } x_1(k+1) = x_2(k)$$

$$x_2(k+1) = -0.3x_1(k) + 1.1x_2(k) + e(k)$$

$$x_3(k+1) = x_3(k) + 0.4y(k)$$

$$y(k) = 4x_2(k)$$

$$e(k) = r(k) - (-0.25x_3(k) + p(k))$$

$$p(k) = 0.4y(k) + x_3(k)$$

$$\Rightarrow e(k) = r(k) + 0.25x_3(k)$$

$$- (0.4(4x_2(k)) + x_3(k))$$

$$= -1.6x_2(k) - 0.75x_3(k) + r(k)$$

$$\text{So } x_1(k+1) = x_2(k)$$

$$x_2(k+1) = -0.3x_1(k) + 1.1x_2(k)$$

$$-1.6x_2(k) - 0.75x_3(k) + r(k)$$

$$= -0.3x_1(k) - 0.5x_2(k) - 0.75x_3(k) + r(k)$$

$$x_3(k+1) = x_3(k) + 0.4(4x_2(k))$$

$$= 1.6x_2(k) + x_3(k)$$

$$y(k) = 4x_2(k)$$

National "Brand" 500 SHEETS FILLER 5 SQUARE
 42-381 50 SHEET EASY-EASE® 5 SQUARE
 42-382 100 SHEET EASY-EASE® 5 SQUARE
 42-389 200 SHEET EASY-EASE® 5 SQUARE
 42-392 100 RECYCLED WHITE 5 SQUARE
 42-393 200 RECYCLED WHITE 5 SQUARE
 Note: all 5 A



$$\dot{x}(k+1) = \begin{bmatrix} 0 & 1 & 0 \\ -0.3 & -0.5 & -0.75 \\ 0 & 1.6 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} r(k)$$

$$y(k) = [0 \ 4 \ 0] x(k)$$

Note: observer canonical form and other realizations are also possible.

#2

$$\dot{x}(t) = \begin{bmatrix} 10 & 11 \\ -20 & -21 \end{bmatrix} x(t) = A x(t)$$

In this case,

$$x(k+1) = \underline{\mathcal{D}}(\tau) x(k), \quad \tau = 0.5 s$$

where $\underline{\mathcal{D}}(t) \Leftrightarrow \underline{\mathcal{D}}(s) = (s\underline{\mathcal{D}} - A)^{-1}$

$$(s\underline{\mathcal{D}} - A)^{-1} = \begin{bmatrix} s-10 & -11 \\ 20 & s+21 \end{bmatrix}^{-1}$$

$$= \frac{1}{(s-10)(s+21) + 220} \begin{bmatrix} s+21 & 11 \\ -20 & s-10 \end{bmatrix}$$

$$= \frac{1}{s^2 + 11s + 10} \begin{bmatrix} s+21 & 11 \\ -20 & s-10 \end{bmatrix}$$

$$= \frac{1}{(s+1)(s+10)} \begin{bmatrix} s+21 & 11 \\ -20 & s-10 \end{bmatrix}$$

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$$= \begin{bmatrix} \frac{s+21}{(s+1)(s+10)} & \frac{11}{(s+1)(s+10)} \\ \frac{-20}{(s+1)(s+10)} & \frac{s-10}{(s+1)(s+10)} \end{bmatrix}$$

Since $\frac{k_1 s + k_2}{(s+1)(s+10)} = \frac{a}{s+1} + \frac{b}{s+10}$

and $a = \left[\frac{k_1 s + k_2}{s+10} \right]_{s=-1} = \frac{k_2 - k_1}{9}$

$$b = \left[\frac{k_1 s + k_2}{s+1} \right]_{s=-10} = \frac{10k_1 - k_2}{9}$$

$$\begin{array}{l} k_1 = 1 \\ k_2 = 21 \end{array} \Rightarrow \frac{s+21}{(s+1)(s+10)} = \frac{20/9}{s+1} - \frac{11/9}{s+10}$$

$$\begin{array}{l} k_1 = 0 \\ k_2 = 11 \end{array} \Rightarrow \frac{11}{(s+1)(s+10)} = \frac{11/9}{s+1} - \frac{11/9}{s+10}$$

$$\begin{array}{l} k_1 = 0 \\ k_2 = -20 \end{array} \Rightarrow \frac{-20}{(s+1)(s+10)} = -\frac{20/9}{s+1} + \frac{20/9}{s+10}$$

$$\begin{array}{l} k_1 = 1 \\ k_2 = -10 \end{array} \Rightarrow \frac{s-10}{(s+1)(s+10)} = \frac{-11/9}{s+1} + \frac{20/9}{s+10}$$

and since $e^{-xt} \leftrightarrow \frac{1}{s+x}, t \geq 0$

$$\Phi(T) = \begin{bmatrix} \frac{20}{9}e^{-T} - \frac{11}{9}e^{-10T} & \frac{11}{9}e^{-T} - \frac{11}{9}e^{-10T} \\ -\frac{20}{9}e^{-T} + \frac{20}{9}e^{-10T} & -\frac{11}{9}e^{-T} + \frac{20}{9}e^{-10T} \end{bmatrix}$$

for $T=0.5$

$$\Phi(0.5) = \begin{bmatrix} 1.3396 & 0.7331 \\ -1.3329 & -0.7263 \end{bmatrix}$$

$$\text{so } x(k+1) = \begin{bmatrix} 1.3396 & 0.7331 \\ -1.3329 & -0.7263 \end{bmatrix} x(k)$$

#3

$$x(k+1) = \begin{bmatrix} 1 & 2 \\ -0.25 & -0.5 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} u(k)$$

$$y(k) = [-1 \quad 2] x(k) + [0 \quad 5] u(k)$$

$$x(N) = \begin{bmatrix} 4 \\ -4 \end{bmatrix} \quad \text{and} \quad x(0) = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$N=2$ at most

For reachability, check the controllability matrix

$e(A, B) = [B \ AB]$ in this case
(since $n=2$)

$$= \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}_{AB}$$

This portion contributes
two lin. indep. columns
so $\text{rank}(e(A, B)) = 2$
(max rank already, no
need to check the AB
part)

So any state can be achieved from any
state 0 in at most two steps, and

$$x(2) - A^2 x(0) = e(A, B) \begin{bmatrix} u(1) \\ u(0) \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ -4 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -0.25 & -0.5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -0.25 & -0.5 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 3 & 2 \\ 1 & 1 & -0.75 & -0.5 \end{bmatrix} \begin{bmatrix} u_1(1) \\ u_2(1) \\ u_1(0) \\ u_2(0) \end{bmatrix}$$

So we need the pseudo inverse of $C(A, B)$
where

$$C^{\#}(A, B) = C^T(A, B) (C(A, B) C^T(A, B))^{-1}$$

$$= C^T(A, B) \begin{bmatrix} 14 & -2.25 \\ -2.25 & 2.8125 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 3 & -0.75 \\ 2 & -0.5 \end{bmatrix} \begin{bmatrix} 0.0820 & 0.0656 \\ 0.0656 & 0.4080 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1475 & 0.4786 \\ 0.0656 & 0.4080 \\ 0.1967 & -0.1092 \\ 0.1311 & -0.0729 \end{bmatrix}$$

So

$$\begin{bmatrix} u_{1(1)} \\ u_{2(1)} \\ u_{1(10)} \\ u_{2(10)} \end{bmatrix} = C^{\#}(A, B) \left(\begin{bmatrix} 4 \\ -4 \end{bmatrix} - \begin{bmatrix} 1 \\ -0.25 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -1.3333 \\ -1.3333 \\ 1 \\ 0.6667 \end{bmatrix}$$

or $u(10) = \begin{bmatrix} 1 \\ 0.6667 \end{bmatrix}, u(1) = \begin{bmatrix} -1.3333 \\ -1.3333 \end{bmatrix}$

#4

$$x(k+1) = \begin{bmatrix} 0.8 & 0.8 \\ 1.1 & 0.4 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u(k)$$

$$y(k) = [1 \ 0] x(k) + [0 \ 1] u(k)$$

$$u(k) = K x(k)$$

$$\Rightarrow x(k+1) = (A + BK) x(k)$$

New poles at $\det(zD - A - BK) = 0$

$$\det \left(\begin{bmatrix} z-0.8 & -0.8 \\ -1.1 & z-0.4 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \right) = 0$$

$$\det \begin{bmatrix} z-0.8-k_{11} & -0.8-k_{12} \\ -1.1-k_{21} & z-0.4-k_{22} \end{bmatrix} = 0$$

$$(z-0.8-k_{11})(z-0.4-k_{22}) - (0.8+k_{12})(1.1+k_{21}) = 0$$

$$\begin{aligned} z^2 - (0.8+k_{11}+0.4+k_{22})z + (0.8+k_{11})(0.4+k_{22}) \\ - (0.8+k_{12})(1.1+k_{21}) = 0 \end{aligned}$$

Desired location $(z-0.1)(z-0.8) = 0$
 $z^2 - 0.9z + 0.08 = 0$

$$\Rightarrow 0.8+k_{11}+0.4+k_{22} = 0.9$$

$$(0.8+k_{11})(0.4+k_{22}) - (0.8+k_{12})(1.1+k_{21}) = 0.08$$

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$$\Rightarrow 0.8+k_{11}+0.4+k_{22} = 0.9$$

$$(0.8+k_{11})(0.4+k_{22}) - (0.8+k_{12})(1.1+k_{21}) = 0.08$$

$$k_{11} + k_{22} = -0.3$$

$$k_{22} = -k_{11} - 0.3$$

$$(0.8 + k_{11})(0.4 - k_{11} - 0.3)$$

$$-(0.8 + k_{12})(1.1 + k_{21}) = 0.08$$

$$(0.8 + k_{11})(0.1 - k_{11}) - (0.8 + k_{12})(1.1 + k_{21}) = 0.08$$

One choice $k_{11} = -0.8 \Rightarrow k_{22} = 0.5$

$$k_{12} = 0 \Rightarrow k_{21} = -1.2$$

$$K = \begin{bmatrix} -0.8 & 0 \\ -1.2 & 0.5 \end{bmatrix}$$