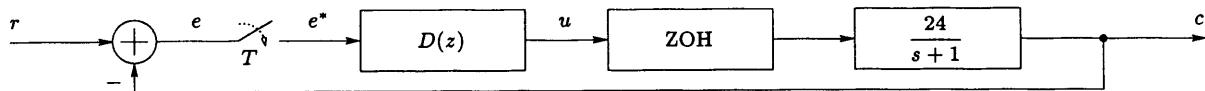


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1. The performance of the following control system is to be improved by a digital controller. Assume the sampling time $T = 0.1$ s.



- (a) Specify the performance of the closed-loop system, when $D(z) = 1$. (10pts)
- (b) A compensator $D(z)$ is selected, such that
- $$u(k) = u(k - 1) + Ke(k) - 0.7Ke(k - 1),$$
- where K is a gain to be specified. Determine this gain, so that the percent overshoot is in the range of 10–25%, and the response is as fast as possible. (15pts)

- (c) Determine the percent overshoot and the 2% settling time for the compensated system. (05pts)
2. A unity-feedback control system is to be designed satisfying the following conditions.

- The 2% settling time is approximately 7 seconds.
- The percent overshoot is about 10% for a unit step input.
- The steady-state error is zero for a unit step input.

The ZOH-Plant combination is described by

$$G(z) = \frac{z + 0.1}{(z - 0.5)(z - 0.9)},$$

where the sampling time $T = 1$ s. Assume the controller

$$D(z) = K \frac{(z - b_1)(z - b_2)}{(z - a_1)(z - a_2)},$$

where b_1 , b_2 , a_1 , and a_2 are all real numbers. Complete the design by specifying K , b_1 , b_2 , a_1 , and a_2 . (35pts)

3. A system is described by

$$\begin{aligned}\mathbf{x}(k+1) &= \begin{bmatrix} 0 & 1 \\ 0.5 & -0.3 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} u(k), \\ y(k) &= [0 \ 1] \mathbf{x}(k),\end{aligned}$$

where u , \mathbf{x} , and y are the input, the state, and the output variables, respectively.

- (a) Determine if the system is reachable and observable. (05pts)
- (b) Find an *open-loop* control sequence which will take the state of the system from $[0 \ 0]^T$ to $[1 \ 0]^T$ in a minimum number of steps and will maintain this state thereafter. (10pts)
- (c) Convert this open-loop control sequence to a state-feedback controller by assuming

$$u(k) = a\mathbf{1}(t) - K\mathbf{x}(k).$$

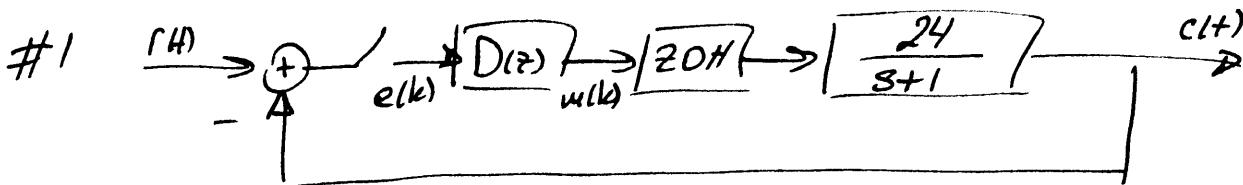
In other words, determine the constant a and K . (10pts)

- (d) Assume that the states are not available, and design an observer with poles at 0.1 and 0.1. Ignore about the fact that the final state may no longer be reached in the same number of steps with the above controller. (10pts)

EE331

Exam #2
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1/2



$$T=0,1$$

$$G_{eq}(z) = (1-z^{-1}) \cancel{2} \left[\frac{24}{s(s+1)} \right] = \frac{z-1}{z} \cancel{2} \left[\frac{24}{s} - \frac{24}{s+1} \right]$$

$$= \frac{2.284}{z-0.905}$$

a,, Closed-loop trans. funct $\frac{C(z)}{R(z)} = \frac{G_{eq}(z)}{1+G_{eq}(z)}$ for $D(z)=1$

$$= \frac{2.284}{z-0.905+2.284} = \frac{2.284}{z+1.379}$$

\uparrow
unstable

b,, $D(z) = \frac{M(z)}{E(z)}$

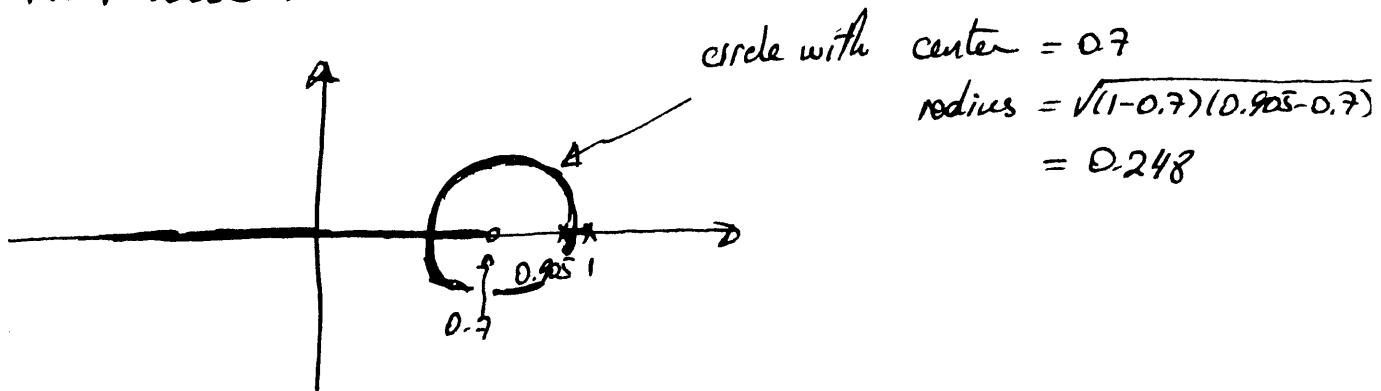
since $Bel(k+1) = 0.7Bel(k) + u(k+1) - u(k)$

$$\beta \cancel{2} E(z) = 0.7\beta \cancel{E(z)} + \cancel{2} M(z) - M(z)$$

$$D(z) = \frac{M(z)}{E(z)} = \frac{\beta(z-0.7)}{(z-1)}$$

Then $D(z) G_{eq}(z) = \frac{2.284 \beta (z-0.7)}{(z-1)(z-0.905)}$

Root-locus is



A careful root locus is on the next page.

From the plot, we realize that the closed-loop poles should be as close as possible to the origin and for those choices α will be positive and for 10-25% overshoot, we need to pick $\xi = 0.9$

$$\text{i.e. } z_d \approx 0.52 \pm j0.16$$

$$\text{or } \left| \frac{2.284 B (z-0.7)}{(z-1)(z-0.905)} \right| = 1 \Rightarrow B = 0.384$$

$$z = 0.52 + j0.16$$

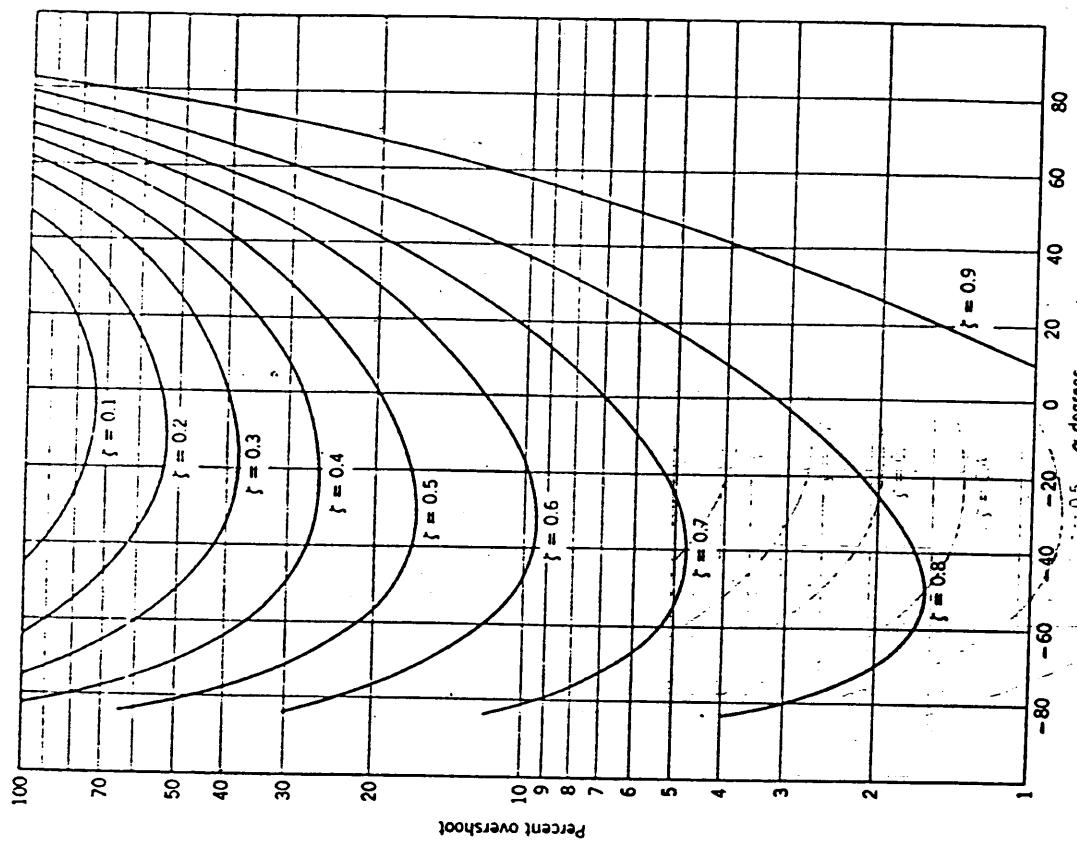


FIGURE 4.7 Step response of quadratic system. Graph of percent overshoot in terms of ξ and α .

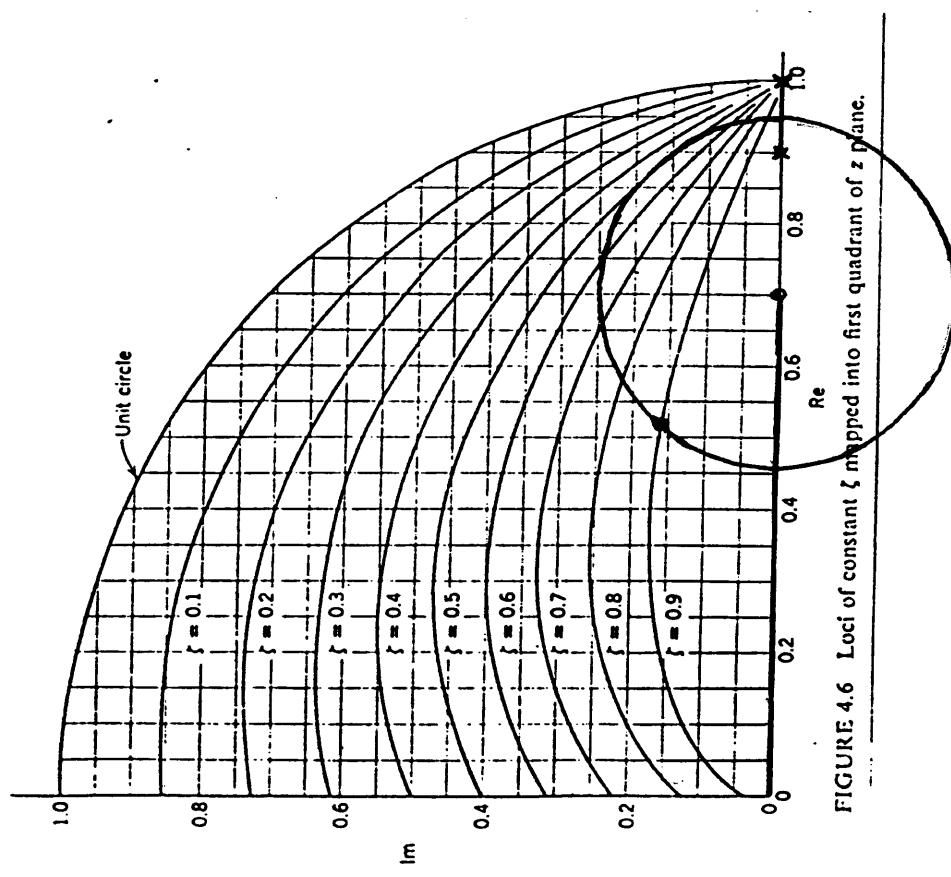


FIGURE 4.6 Loci of constant ξ mapped into first quadrant of z plane.

$$c_{11} \quad \alpha = \frac{\pi}{2} + \tan^{-1} \frac{\omega_p}{1-\sigma_p} - \tan^{-1} \frac{\omega_p}{\sigma_2 - \sigma_p}$$

$$= 66.8^\circ$$

From the other graph : overshoot $\approx 20\%$ for $\xi = 0.9$

The 2% settling time from

$$e^{(N-1)} \leq 0.02$$

$$(N-1) \ln \varphi \leq \ln 0.02$$

$$N \geq \frac{\ln 0.02}{\ln \varphi} + 1 = \frac{\ln 0.02}{\ln \sqrt{(0.52)^2 + (0.16)^2}} + 1 = 7.43$$

i.e. $N = 8$

#2 Unity Feedback system

$$G(z) = \frac{K(z+0.1)}{(z-0.5)(z-0.9)} \quad T=1$$

$$D(z) = \frac{(z-b_1)(z-b_2)}{(z-a_1)(z-a_2)}$$

Steady-state error zero for a step input \Rightarrow System is of TYPE I \Rightarrow One of the poles is 1.

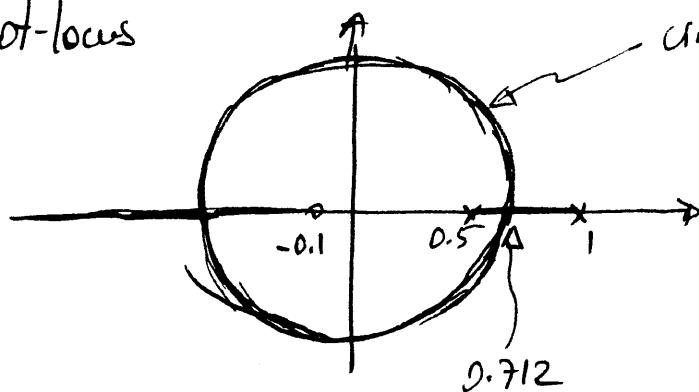
let $a_1 = 1$ and $D(z) = D_1(z)D_2(z)$

where $D_1(z) = \frac{z-0.9}{z-1}$

for convenience and for not to increase the system order

i.e. $D_1(z)G(z) = \frac{K(z+0.1)}{(z-0.5)(z-1)}$

Root-locus



circle with center = -0.1

$$\text{radius} = \sqrt{(0.5+0.1)(1+0.1)} \\ = 0.812$$

2% settling time = 7 sec $\Rightarrow NT=7$ or $N=7$

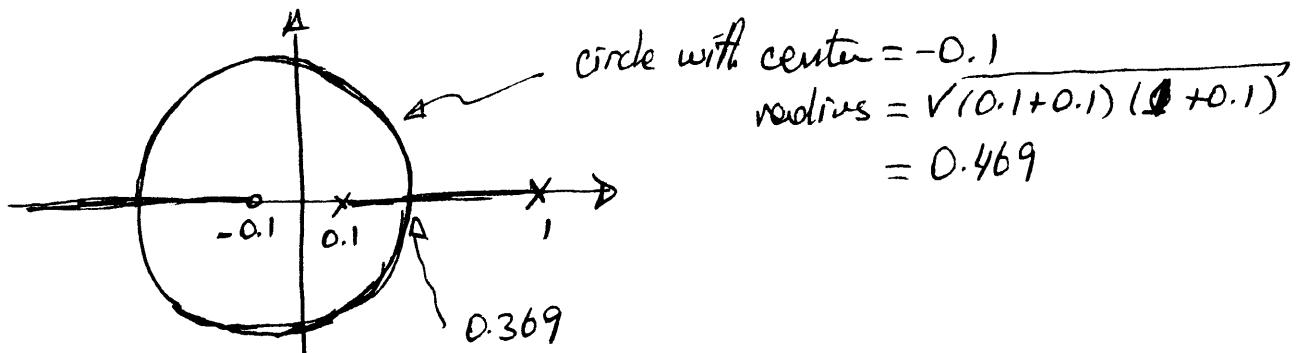
and $\rho^{7-1} \leq 0.02$

$$\rho \leq (0.02)^{\frac{1}{7-1}} = 0.521$$

However, the root-loci do not even get inside this region. We need to speed up the system. One way is to cancel the pole at 0.5 and put a faster pole?

let $D_2(z) = \frac{z-0.5}{z-0.1}$

then $D(z)G(z) = \frac{K(z+0.1)}{(z-1)(z-0.1)}$



A detailed root-locus is given on the next page.

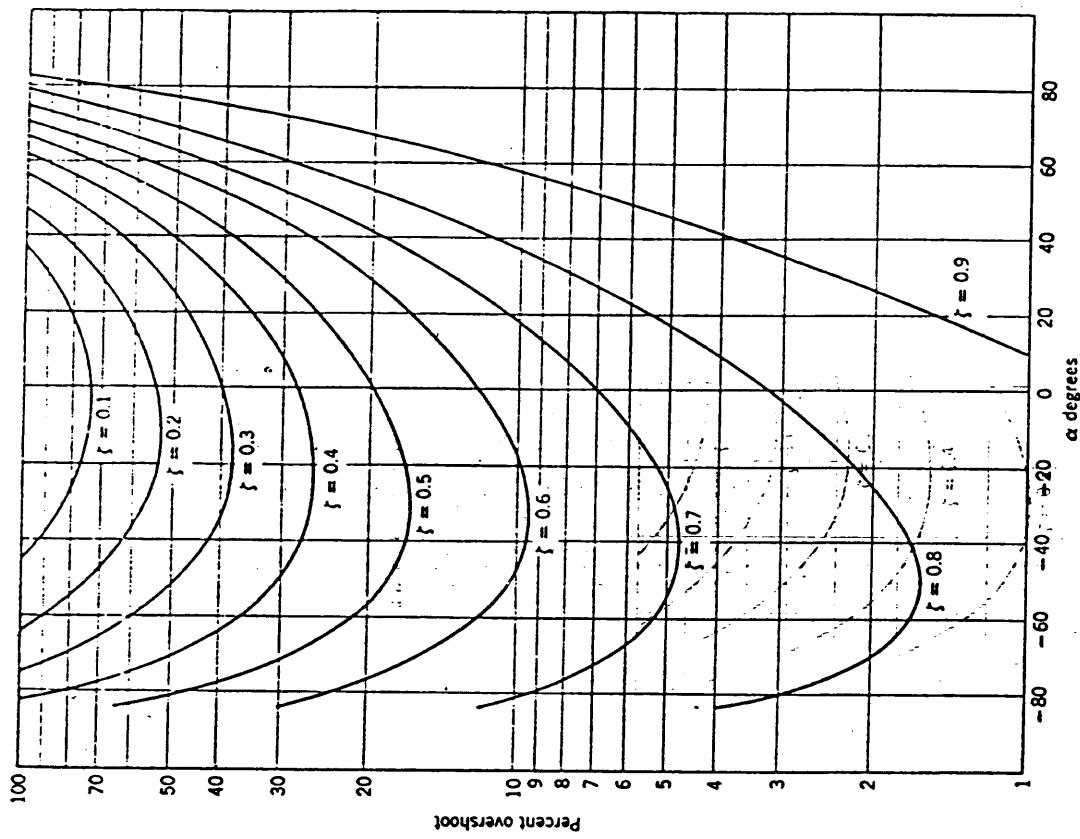


FIGURE 4.7 Step response of quadratic system. Graph of percent overshoot in terms of ξ and α .

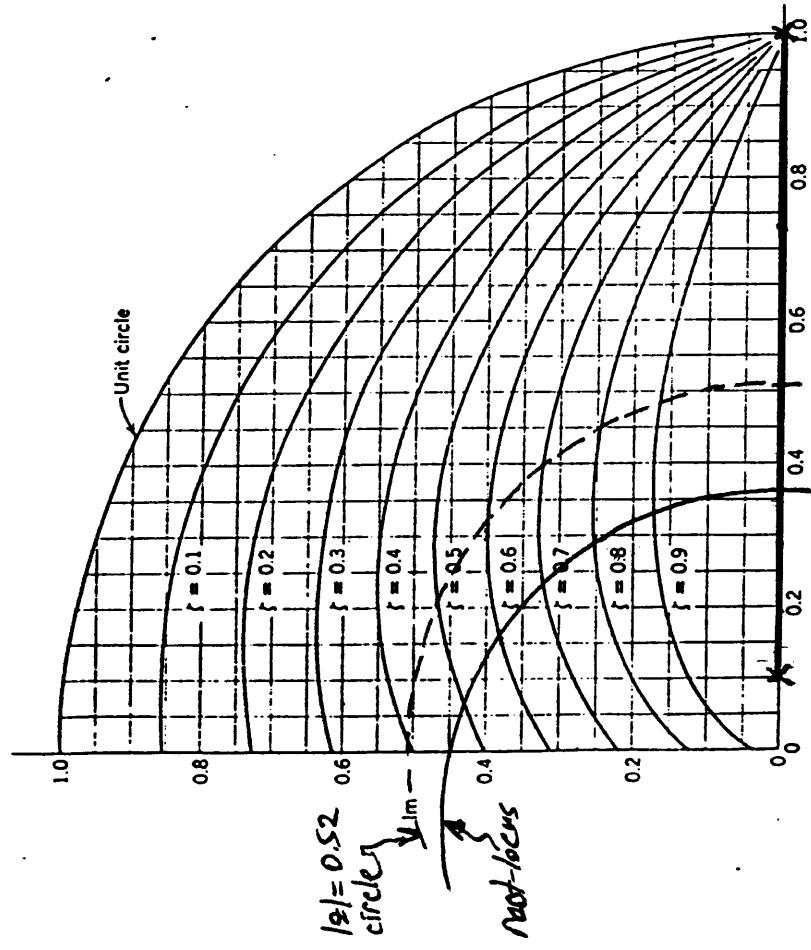


FIGURE 4.6 Loci of constant ξ mapped into first quadrant of z plane.

since 10% overshoot is required and $\alpha > 0$

let $\xi = 0.6 \Rightarrow z_d \approx 0.16 \pm j0.38$ from the graph

\hookrightarrow gives $\alpha = 15.4^\circ$
and a little bit more
than 10% overshoot

and

$$\left| \frac{K(z+0.1)}{(z-1)(z-0.1)} \right|_{z=z_d} = 1 \Rightarrow K = 0.77$$

#3

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ 0.5 & -0.3 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ -0.5 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(k)$$

a) Reachable if $C(B, P) = \begin{bmatrix} B^T P & P \end{bmatrix}$ has full rank

$$= \begin{bmatrix} -0.5 & 1 & 0 \\ 0.15 & 1 & -0.5 \end{bmatrix} \leftarrow \text{has rank } = 2$$

so REACHABLE

Observable if $D(C, B) = \begin{bmatrix} C^T B \\ C \end{bmatrix}$ has full rank

$$= \begin{bmatrix} 0.5 & -0.3 \\ 0 & 1 \end{bmatrix} \leftarrow \text{has rank } = 2$$

so OBSERVABLE

$$b), x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} ; x(N) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

for minimum time

let $N=1$

$$x(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0.5 & -0.3 \end{bmatrix} x(0) + \begin{bmatrix} 0 \\ -0.5 \end{bmatrix} u(0)$$

so $\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix} u(0)$; cannot be satisfied,
i.e., $N \neq 1$

let $N=2$ (has to work since the system is of order 2 and controllable)

$$\text{then } x(1) = \underline{\oplus} x(0) + \underline{\Gamma}_{u(0)}$$

$$x(2) = \underline{\oplus} x(1) + \underline{\Gamma}_{u(1)}$$

$$= \underline{\oplus} [\underline{\oplus} x(0) + \underline{\Gamma}_{u(0)}] + \underline{\Gamma}_{u(1)}$$

$$= \underline{\oplus}^2 x(0) + \underline{\oplus} \underline{\Gamma}_{u(0)} + \underline{\Gamma}_{u(1)}$$

$$\text{or } \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \underline{\oplus}^2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \left[\begin{array}{cc} \underline{\oplus} & \underline{\Gamma} \\ \underline{\Gamma} & 1 \end{array} \right] \begin{bmatrix} u(0) \\ u(1) \end{bmatrix}$$

\uparrow
 $C(\underline{\oplus}, \underline{\Gamma})$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.5 & 1 \\ 0.15 & 1 \end{bmatrix} \begin{bmatrix} u(0) \\ u(1) \end{bmatrix}$$

$$\text{and } \begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \frac{1}{(0.5)^2} \begin{bmatrix} -0.5 & 0 \\ -0.15 & -0.5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -0.6 \end{bmatrix}$$

To maintain this state $x(2) = x(3) = \dots = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\text{or } x(3) = \underline{\oplus} x(2) + \underline{\Gamma}_{u(2)}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0.5 & -0.3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -0.5 \end{bmatrix} u(2)$$

$$\begin{bmatrix} 1 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix} u(0) \quad \text{not possible to keep the state fixed, but can come back to } \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ in two steps.}$$

similar to th previous derivation

$$\begin{matrix} \mathbf{x}(4) - \cancel{\mathbf{D}}^2 \mathbf{x}(2) \\ \downarrow \\ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{matrix} = \left[\begin{matrix} \cancel{\mathbf{D}}^2 \mathbf{P} & \mathbf{P} \\ \mathbf{P} & 0 \end{matrix} \right] \begin{bmatrix} u(2) \\ u(3) \end{bmatrix}$$

$$\text{or } \begin{bmatrix} u(2) \\ u(3) \end{bmatrix} = \frac{1}{(0.5)^2} \begin{bmatrix} -0.5 & 0 \\ -0.15 & -0.5 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.15 \end{bmatrix} = \begin{bmatrix} -1 \\ -0.6 \end{bmatrix}$$

$$\text{so } u(k) = \{-2, -0.6, \underbrace{-1, -0.6, \dots}_{\text{repeats}}\}$$

$$\text{or } u(k) = \{-2, \underbrace{-0.6, -1, \dots}_{\text{repeats}}\}$$

$$c_{11} \quad \text{let } u(k) = A - K_1 x_1(k) - K_2 x_2(k) \quad ; k \geq 0$$

$$k=0 ; \quad u(0) = A - K_1 \cancel{x_1(0)}^0 - K_2 \cancel{x_2(0)}^0 \\ -2 = A$$

$$k=1 ; \quad u(1) = -2 - K_1 x_1(1) - K_2 x_2(1)$$

$$\text{need to determine } x(1) = \cancel{\mathbf{D}}^2 \mathbf{x}(0) + \mathbf{P} u(0)$$

$$= \begin{bmatrix} 0 & 1 \\ 0.5 & -0.3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -0.5 \end{bmatrix} u(0)$$

$$x(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\text{so } -0.6 = -2 - K_1 \cdot 0 - K_2 \cdot 1 \Rightarrow K_2 = -1.4$$

$$k=2 : u(2) = -2 - k_1 \overset{1}{x_1(2)} + \overset{0}{1.4 x_2(2)}$$

$$-1 = -2 - k_1 \Rightarrow k_1 = -1$$

$$\therefore u(k) = -2 \mathbf{1}(k) - [-1 \quad -1.4] \mathbf{x}(k)$$

Note: These $u(k)$'s match the open-loop sequence, since they match the repeated portion.

d,, char. poly of the obs

$$\begin{aligned} &= \det (zI - \cancel{B} + LC) \\ &= \det \left\{ \begin{bmatrix} z & -1 \\ -0.5 & z+0.3 \end{bmatrix} + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} [0 \quad 1] \right\} \\ &= z^2 + (0.3 + l_2)z + 0.5(l_1 - 1) \end{aligned}$$

$$\begin{aligned} \text{Desired char. poly. of the obs} &= (z - 0.1)^2 \\ &= z^2 - 0.2z + 0.01 \end{aligned}$$

or term by term comparison

$$\begin{aligned} 0.3 + l_2 &= -0.2 \Rightarrow l_2 = -0.5 \\ &= 0.01 \Rightarrow l_1 = 1.02 \end{aligned}$$

i.e. $L = [1.02 \quad -0.5]$