EE 337

Exam#1

75 minutes

Nov. 07, 2000

1. Consider the following sets of desired associations.

$$\mathbf{a}_1 = \begin{bmatrix} -1 & 1 & 1 & -1 \end{bmatrix}^T \longleftrightarrow \begin{bmatrix} 1 & -1 \end{bmatrix}^T = \mathbf{b}_1,$$

and

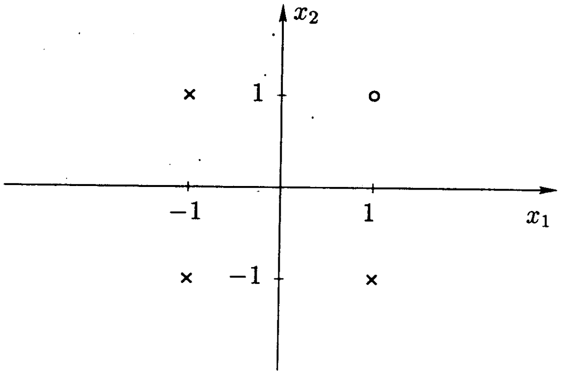
$$\mathbf{a}_2 = \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}^T \longleftrightarrow \begin{bmatrix} -1 & 1 \end{bmatrix}^T = \mathbf{b}_2.$$

   (a) Design a correlation matrix memory network using a non-iterative method, where the association is approximated by the product correlation. (05pts)

   (b) Design a correlation matrix memory network using a non-iterative method, where the association is optimal. (10pts)

   (c) Apply the input vector $\mathbf{a} = \begin{bmatrix} -1 & 1 & 1 & 1 \end{bmatrix}^T$ to either of the networks designed above, and determine its associated vector $\mathbf{b}$. Compare vector $\mathbf{b}$ with $\mathbf{b}_1$ and $\mathbf{b}_2$ as vector $\mathbf{a}$ compares with $\mathbf{a}_1$ and $\mathbf{a}_2$, and explain the comparison results. (10pts)

2. The data that are marked on the following figure need to be classified by the use of a perceptron, such that the circle is represented by 1 and the crosses by -1 at the output.



   (a) Design a non-iterative classifier network, assuming that the activation function is the sign function. Plot the decision surface. (10pts)

   (b) In another network that is to use an iterative method to determine the weights, assume that the initial weights are given as

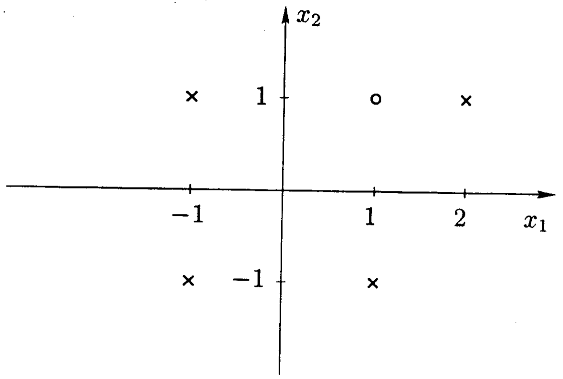
$$\mathbf{w} = \begin{bmatrix} w_0(0) & w_1(0) & w_2(0) \end{bmatrix}^T = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}^T.$$

   Note here that w_0 corresponds to the bias input.

      i. Plot the decision surface based on the initial weights, and determine the output error for each of the training inputs assuming that the activation function is the sign function. (05pts)

      ii. Update the weights once for each of the incorrectly classified points using the delta rule with the batch learning method, when the learning rate $\eta = 0.25$. Plot the decision surface based on the updated weights. (10pts)

3. The data that are marked on the following figure need to be classified by the use of perceptrons, such that the circle is represented by 1 and the crosses by -1 at the output.

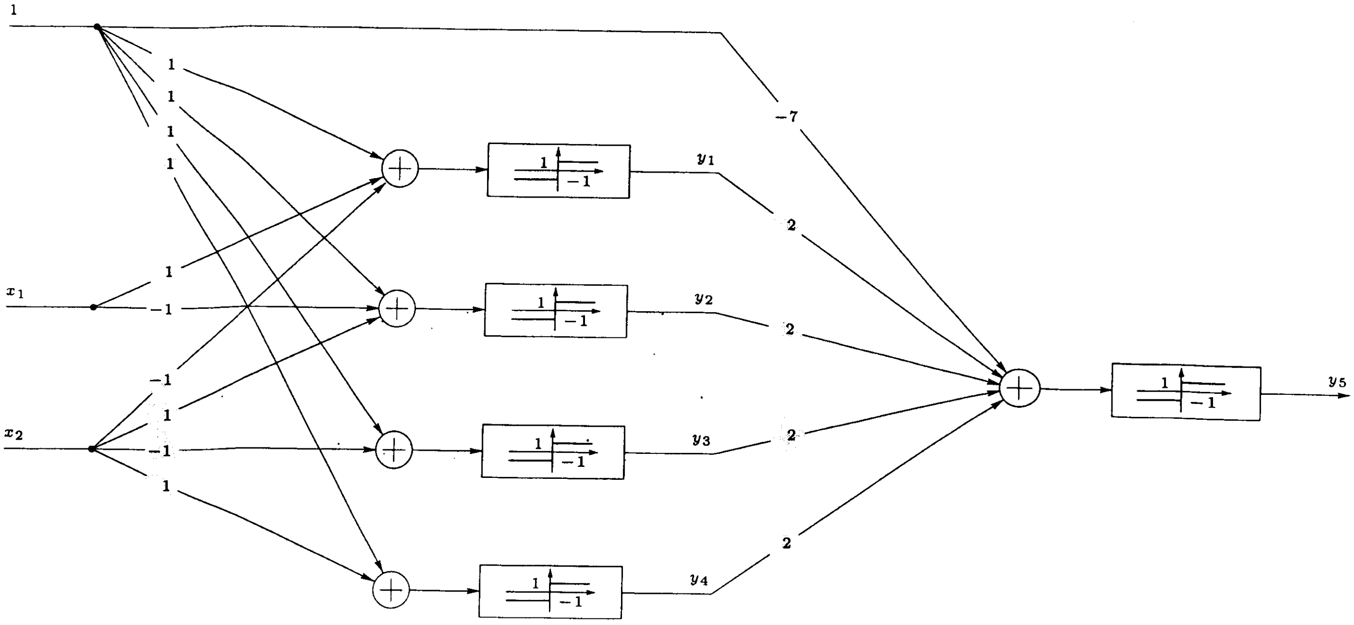


Design a non-iterative classifier network, assuming that the activation functions are the sign function. Plot all the decision surfaces.

(25pts)

4. Given the network below, determine and plot the region for $y_5 = 1$ on the x_1-x_2 plane. Show all your work.

(25pts)



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Solutions

1. Consider the following sets of desired associations.

$$\mathbf{a}_1 = \begin{bmatrix} -1 & 1 & 1 & -1 \end{bmatrix}^T \longleftrightarrow \begin{bmatrix} 1 & -1 \end{bmatrix}^T = \mathbf{b}_1,$$

and

$$\mathbf{a}_2 = \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}^T \longleftrightarrow \begin{bmatrix} -1 & 1 \end{bmatrix}^T = \mathbf{b}_2.$$

(a) Design a correlation matrix memory network using a non-iterative method, where the association is approximated by the product correlation.

Solution: When the association is approximated by the product correlation, the weight matrix M_{product} of the correlation matrix memory network is given by

$$M_{\text{product}} = BA^T = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 \end{bmatrix}\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix},$$

or

$$M_{\text{product}} = \begin{bmatrix} -2 & 0 & 2 & 0 \\ 2 & 0 & -2 & 0 \end{bmatrix}.$$

(b) Design a correlation matrix memory network using a non-iterative method, where the association is optimal.

Solution: When the association is optimal, the weight matrix M_{optimal} of the correlation matrix memory network is given by

$$\begin{aligned} M_{\text{optimal}} &= BA^{\#} = B(A^TA)^{-1}A^T \\ &= \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 \end{bmatrix}\left(\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 \end{bmatrix}^T\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 \end{bmatrix}\right)^{-1}\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 \end{bmatrix}^T \\ &= \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 \end{bmatrix}\begin{bmatrix} \mathbf{a}_1^T\mathbf{a}_1 & \mathbf{a}_1^T\mathbf{a}_2 \\ \mathbf{a}_2^T\mathbf{a}_1 & \mathbf{a}_2\mathbf{a}_2^T \end{bmatrix}^{-1}\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 \end{bmatrix}^T \\ &= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}^{-1}\begin{bmatrix} -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \\ &= \frac{1}{4}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}, \end{aligned}$$

where $(\cdot)^{\#}$ is the pseudo inverse. So,

$$M_{\text{optimal}} = \begin{bmatrix} -1/2 & 0 & 1/2 & 0 \\ 1/2 & 0 & -1/2 & 0 \end{bmatrix}.$$

(c) Apply the input vector $\mathbf{a} = \begin{bmatrix} -1 & 1 & 1 & 1 \end{bmatrix}^T$ to either of the networks designed above, and determine its associated vector $\mathbf{b}$. Compare vector $\mathbf{b}$ with $\mathbf{b}_1$ and $\mathbf{b}_2$ as vector $\mathbf{a}$ compares with $\mathbf{a}_1$ and $\mathbf{a}_2$, and explain the comparison results.

Solution: Using the weight matrix M_{product}, we get

$$\mathbf{b}_{\text{product}} = M_{\text{product}}\mathbf{a} = \begin{bmatrix} -2 & 0 & 2 & 0 \\ 2 & 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix} = 4\mathbf{b}_1.$$

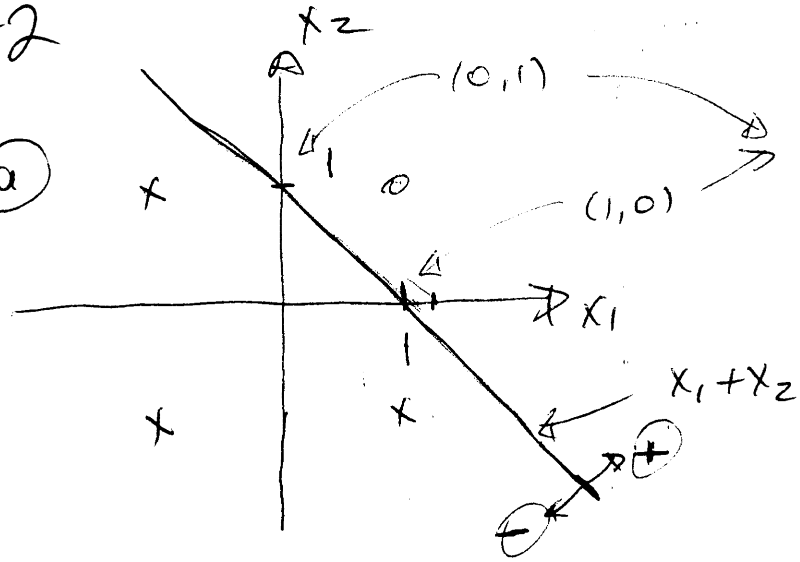
Similarly, using the weight matrix M_{optimal}, we get

$$\mathbf{b}_{\text{optimal}} = M_{\text{optimal}}\mathbf{a} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \mathbf{b}_1.$$

In other words, even though $\mathbf{a} \neq \mathbf{a}_1$, we get $\mathbf{b} = \mathbf{b}_1$. This result displays the error correcting capability of the memory networks, where any given input provides the output of the memorized input with the smallest hamming distance.

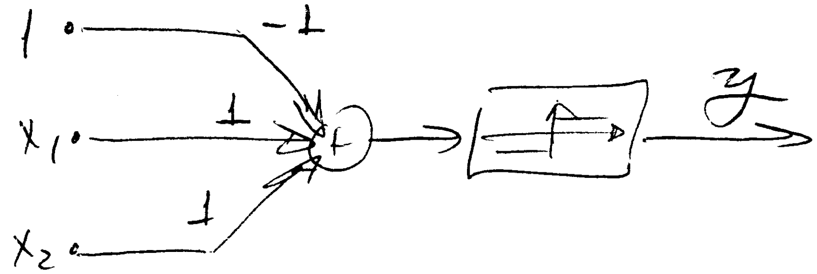
#2

(a)



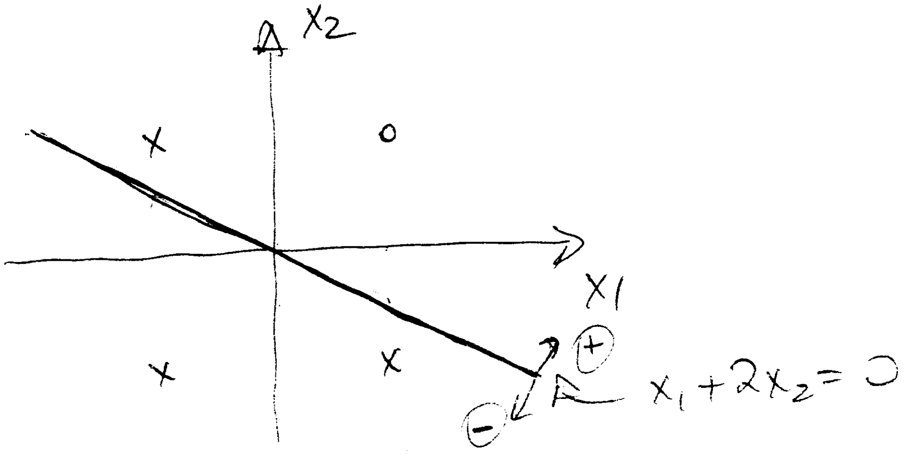
$$x_1 - 0 = \frac{1-0}{0-1}(x-1)$$

$$x_1 = -x+1$$



(b) $\underline{w} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \Rightarrow 0 \cdot 1 + 1 \cdot x_1 + 2 \cdot x_2 = 0$

$x_1 + 2x_2 = 0$



$$\underline{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \underline{w}^T\underline{x}_1 = 3 \Rightarrow y = 1,\ e = 0$$

$$\underline{x}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \Rightarrow \underline{w}^T\underline{x}_2 = 1 \Rightarrow y = 1,\ e = -2$$

$$\underline{x}_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \Rightarrow \underline{w}^T\underline{x}_3 = -3 \Rightarrow y = -1,\ e = 0$$

$$\underline{x}_4 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow \underline{w}^T\underline{x}_4 = -1 \Rightarrow y = -1,\ e = 0$$

$$\Delta\underline{w} = \eta(y_d - y)\underline{x} \quad \text{and} \quad \Delta\underline{w}\Big|_{\substack{\underline{x}=\underline{x}_1 \\ \underline{x}=\underline{x}_3 \\ \underline{x}=\underline{x}_4}} = 0$$

$$\Delta\underline{w}\Big|_{\underline{x}=\underline{x}_2} = 0.25\,(-1-1)\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \\ -1/2 \end{bmatrix}$$

$$\underline{w}(+) = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} -1/2 \\ 1/2 \\ -1/2 \end{bmatrix} = \begin{bmatrix} -1/3 \\ 3/2 \\ 3/2 \end{bmatrix}$$

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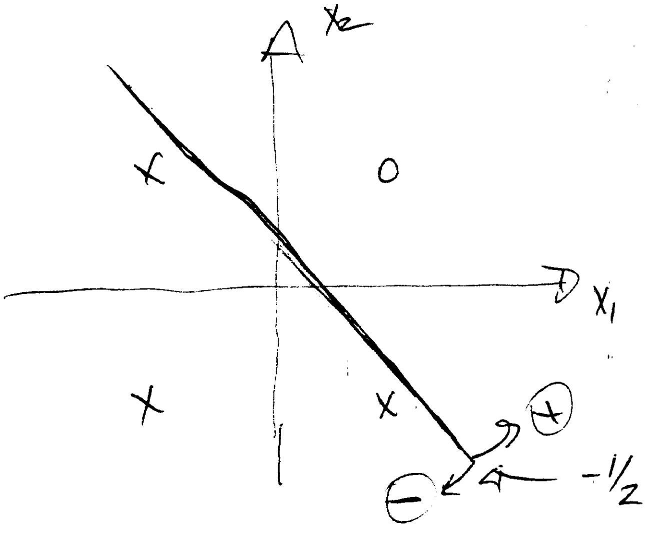
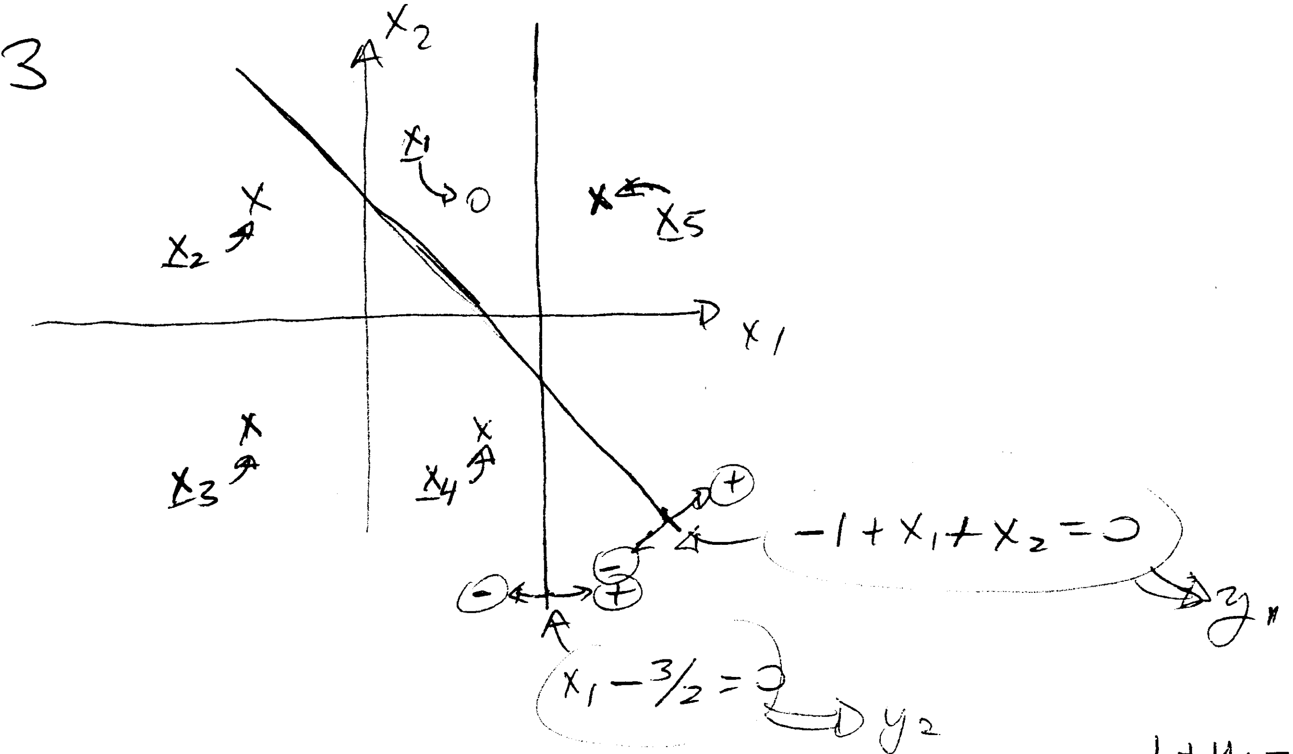


$-\frac{1}{2} + \frac{3}{2}x_1 + \frac{3}{2}x_2 = 0$

or $-1 + 3x_1 + 3x_2 = 0$

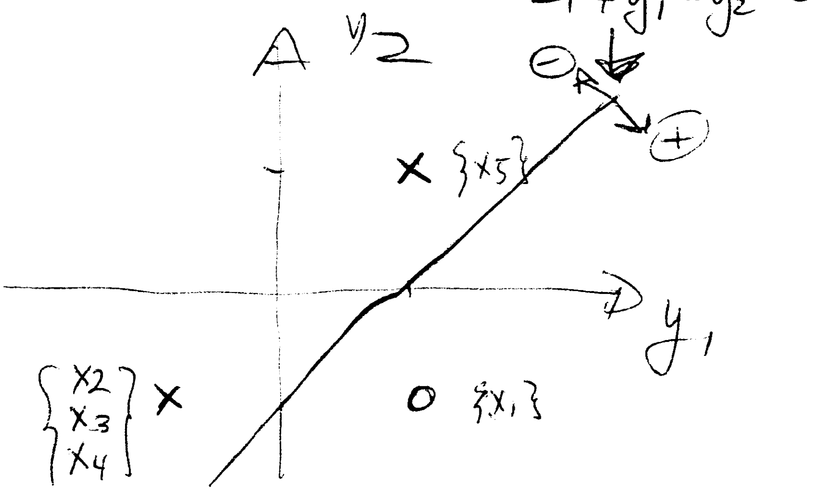
#3



$-1 + x_1 + x_2 = 0$ → y_1

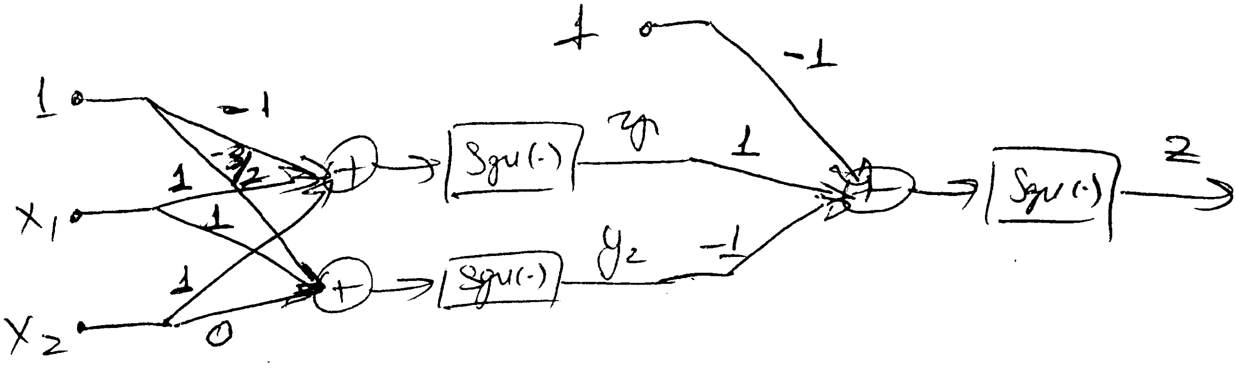
$x_1 - \frac{3}{2} = 0$ → y_2

x_i	(y_1, y_2)
$\underline{x}_1$	$(1, -1)$
$\underline{x}_2$	$(-1, -1)$
$\underline{x}_3$	$(-1, -1)$
$\underline{x}_4$	$(-1, -1)$
$\underline{x}_5$	$(1, 1)$



$-1 + y_1 - y_2 = 0$

$$\text{so} \quad z = \operatorname{sgn}(-1 + y_1 - y_2)$$



#4 $y_1 = \operatorname{sgn}(1 + x_1 - x_2)$

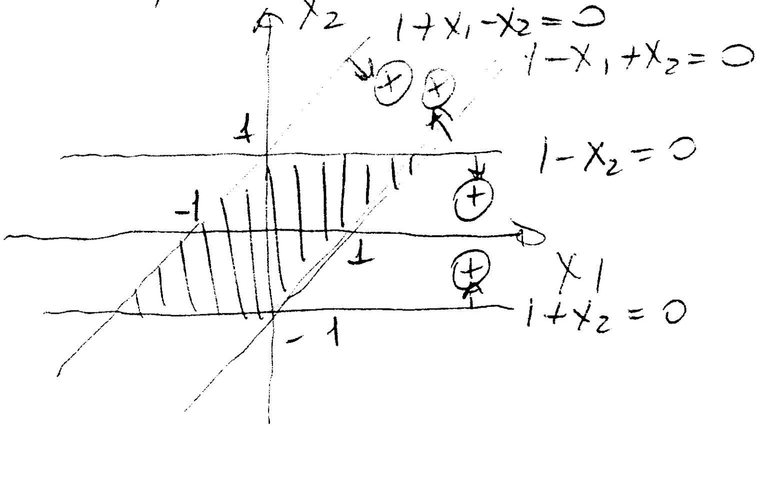
$y_2 = \operatorname{sgn}(1 - x_1 + x_2)$

$y_3 = \operatorname{sgn}(1 - x_2)$

$y_4 = \operatorname{sgn}(1 + x_2)$

$$y_5 = \operatorname{sgn}(-7 + 2y_1 + 2y_2 + 2y_3 + 2y_4)$$

$$= \begin{cases} 1, & \text{if } y_1 = y_2 = y_3 = y_4 = 1; \\ -1, & \text{otherwise.} \end{cases}$$


x_2
1+x_1-x_2=0
1-x_1+x_2=0
+
+
1
1-x_2=0
+
-1
1
x_1
+
1+x_2=0
-1