

1. Some of the neural network learning methods, including the Back-Propagation method, use derivatives of the activation functions. Determine the derivative,  $S'(x)$  of the activation function

$$S(x) = \frac{2}{1 + e^{-cx}} - 1,$$

and express it in terms of  $S(x)$ .

(10pts)

2. Consider a Bivalent Bidirectional Associative Memory (BAM) network with a connection matrix

$$M = \begin{bmatrix} -2 & 0 & 2 \\ 0 & -2 & 0 \\ 2 & 0 & -2 \\ 0 & 2 & 0 \end{bmatrix},$$

and with bivalent zero-threshold activation functions.

- (a) Start with the pair  $A_1 = [1, 0, 1, 1]$  and  $B_1 = [1, 0, 0]$ ; and determine one of the fixed points,  $(A_1^*, B_1^*)$  of the network. (10pts)
  - (b) Using only the information obtained in the previous part, write down as many fixed points of the network as possible. (05pts)
3. Given the following two sets of desired associations,

$$A_1 = [1, 0, 0] \longleftrightarrow B_1 = [0, 1],$$

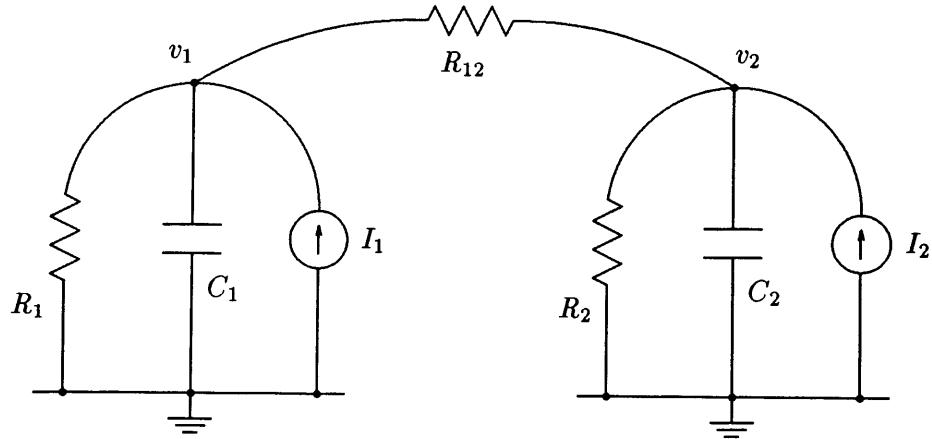
and

$$A_2 = [0, 0, 1] \longleftrightarrow B_2 = [1, 0],$$

determine the Bidirectional Associative Memory (BAM) connection matrix with

- (a) Bipolar Outer-Product Learning, and (10pts)
- (b) (Bipolar) Optimal Linear Association. (15pts)

4. A simple, two-neuron, linear, continuous-time Hopfield network is given below.



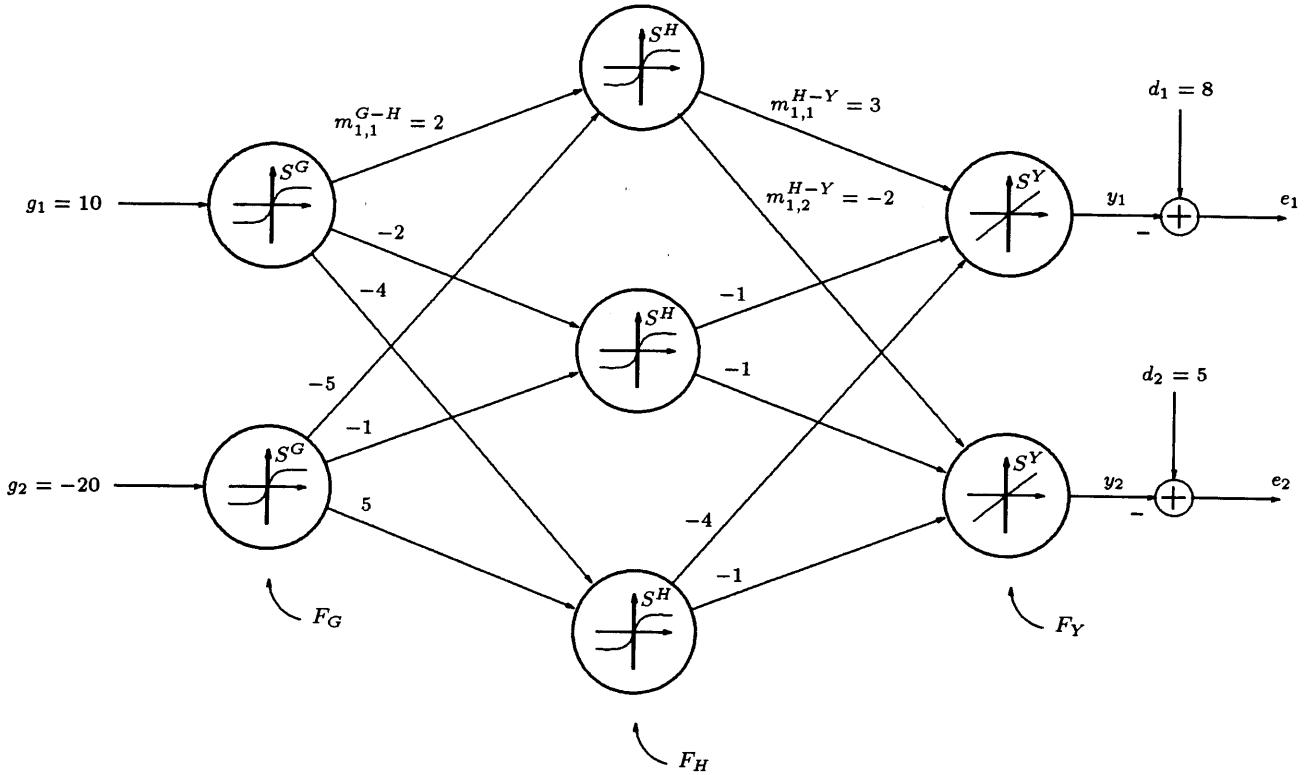
Let  $I_1 = 1$ , and  $I_2 = -1$ . Determine a set of resistors  $R_1$ ,  $R_{12}$ , and  $R_2$ , such that the Hopfield network memorizes

$$[v_1, v_2] = [5, -10],$$

i.e., the network settles to these values for all initial conditions. (20pts)

*Hint:* In the expression of the steady-state solutions for  $v_1$  and  $v_2$ , let  $\alpha = R_{12}/(R_1 + R_{12} + R_2)$ , then determine  $R_1$  and  $R_2$  in terms of  $\alpha$ ; and from the definition of  $\alpha$ , obtain a proper  $R_{12} (> 0)$  for a proper choice of  $\alpha$ .

5. The last three layers of a feedforward neural network is shown below.



The numbers given above the connections are the old connection weights. The activations in the  $F_G$ ,  $F_H$ , and  $F_Y$  fields are

$$S^G(g_r) = \frac{2}{1 + e^{-100g_r}} - 1,$$

$$S^H(h_q) = \frac{2}{1 + e^{-10h_q}} - 1,$$

and

$$S^Y(y_j) = y_j,$$

respectively. The neural network is to be trained using the backpropagation method.

- (a) Determine the signal before and after each neuron, and the errors at the output. (10pts)
- (b) Determine the updated values for the weights  $m_{1,1}^{H-Y}$  and  $m_{1,1}^{H-Y}$ . (10pts)
- (c) Determine the updated value for the weight  $m_{1,1}^{G-H}$ . (10pts)

#1

$$S(x) = \frac{2}{1+e^{-cx}} - 1$$

$$S'(x) = -\frac{-ce^{-cx} \cdot 2}{(1+e^{-cx})^2} = \frac{ce^{-cx}}{1+e^{-cx}} \left( \frac{2}{1+e^{-cx}} \right)$$

A  $S(x)+1$

$$= c \left( \frac{-1+1+e^{-cx}}{1+e^{-cx}} \right) (S(x)+1)$$

$$= c \left( -\frac{1}{1+e^{-cx}} + 1 \right) (S(x)+1)$$

$$= -\frac{c}{2} \left( \frac{2}{1+e^{-cx}} - 2 \right) (S(x)+1)$$

$$= -\frac{c}{2} (S(x)+1 - 2) (S(x)+1)$$

$$= -\frac{c}{2} (S(x)-1) (S(x)+1)$$

$$= -\frac{c}{2} (S(x)^2 - 1)$$

$$= \frac{c}{2} (1 - S(x)^2)$$

## #2 Bivalent BAM

$$M = \begin{bmatrix} -2 & 0 & 2 \\ 0 & -2 & 0 \\ 2 & 0 & -2 \\ 0 & 2 & 0 \end{bmatrix}$$

a,,  $A_1 = [1, 0, 1, 1]$        $B_1 = [1, 0, 0]$

$$\underline{A_1 = S(B_1, M^T)} \quad \underline{A_1 M} \quad \underline{B_1 = S(A_1, M)} \quad \underline{B_1 M^T}$$

$$[1, 0, 1, 1] \downarrow$$

$$[1, 0, 0]$$

$$[1, 0, 1, 1] \rightarrow [0, 2, 0] \rightarrow [1, 1, 0] \rightarrow [-2, 0, 2, 0]$$

$$[0, 0, 1, 1] \rightarrow [2, 2, -2] \rightarrow [1, 1, 0] \rightarrow [-2, -2, 2, 2]$$

$$[0, 0, 1, 1] \rightarrow [2, 2, -2] \rightarrow [1, 1, 0]$$

so  $A_1^* = [0, 0, 1, 1] \leftrightarrow B_1 = [1, 1, 0]$

b,, from part a  $A_1^* = [0, 0, 1, 1] \leftrightarrow B_1^* = [1, 1, 0]$

Its complement  
is also a fixed pt.  $A_1^{*c} = [1, 1, 0, 0] \leftrightarrow B_1^{*c} = [0, 0, 1]$

Also

$$A_2^* = [0, 0, 0, 0] \leftrightarrow B_2^* = [0, 0, 0]$$

and

$$A_2^{*c} = [1, 1, 1, 1] \leftrightarrow B_2^{*c} = [1, 1, 1]$$

are always fixed pts.

#3

$$A_1 = [1, 0, 0] \leftrightarrow B_1 = [0, 1]$$

$$A_2 = [0, 0, 1] \leftrightarrow B_2 = [1, 0]$$

$$A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow X = \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow Y = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$\uparrow$  binary       $\uparrow$  bipolar equivalent

a,, Bipolar Outer-Product learning

$$M = X^T Y = \begin{bmatrix} 1 & -1 \\ -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 0 & 0 \\ 2 & -2 \end{bmatrix}$$

b,, Bipolar Optimal Linear Assoc.

$$M = X^{\#} Y$$

$$X^{\#} = X^T (X X^T)^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & -1 \\ -1 & 1 \end{bmatrix} \left( \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & -1 \\ -1 & 1 \end{bmatrix} \right)^{-1}$$

$$= \begin{bmatrix} 1 & -1 \\ -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}^{-1}$$

since  $X$  has full rank

$$= \begin{bmatrix} 1 & -1 \\ -1 & -1 \\ -1 & 1 \end{bmatrix} \frac{1}{(3)(3) - (-1)(-1)} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

EE301B

EXHIBIT 1  
SOLUTIONS

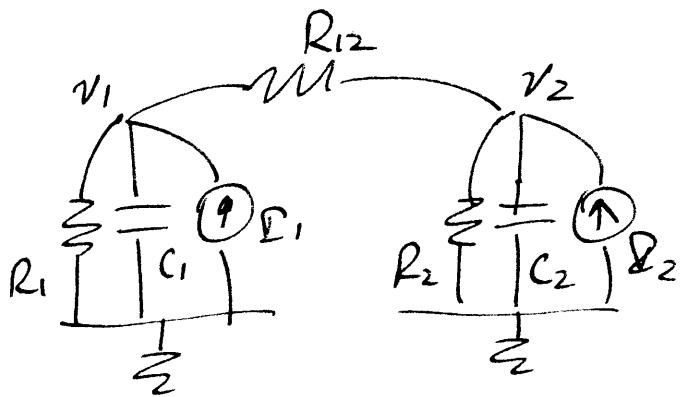
Fall 92 4/8

$$X^{\#} = \frac{1}{8} \begin{bmatrix} 2 & -2 \\ -4 & -4 \\ -2 & 2 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & -1 \\ -2 & -2 \\ -1 & 1 \end{bmatrix}$$

$$M = X^{\#} Y = \frac{1}{4} \begin{bmatrix} 1 & -1 \\ -2 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -2 & 2 \\ 0 & 0 \\ 2 & -2 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -1 & 1 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}$$

#4



Current law at the nodes

$$\frac{v_1}{i_{SC_1}} + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_{12}} = \underline{\Omega}_1$$

$$\frac{v_2}{i_{SC_2}} + \frac{v_2}{R_2} + \frac{v_2 - v_1}{R_{12}} = \underline{\Omega}_2$$

$$sC_1v_1 + \left(\frac{1}{R_1} + \frac{1}{R_{12}}\right)v_1 - \frac{1}{R_{12}}v_2 = \underline{\Omega}_1$$

$$sC_2v_2 + \left(\frac{1}{R_2} + \frac{1}{R_{12}}\right)v_2 - \frac{1}{R_{12}}v_1 = \underline{\Omega}_2$$

or  $C_1\dot{v}_1 + \left(\frac{1}{R_1} + \frac{1}{R_{12}}\right)v_1 - \frac{1}{R_{12}}v_2 = \underline{\Omega}_1$

$$C_2\dot{v}_2 + \left(\frac{1}{R_2} + \frac{1}{R_{12}}\right)v_2 - \frac{1}{R_{12}}v_1 = \underline{\Omega}_2$$

at steady state  $v_1$  and  $v_2$  <sup>are</sup> constants  $\Rightarrow \dot{v}_1 = \dot{v}_2 = 0$

or  $\left(\frac{1}{R_1} + \frac{1}{R_{12}}\right)v_1 - \frac{1}{R_{12}}v_2 = \underline{\Omega}_1$

$$\left(\frac{1}{R_2} + \frac{1}{R_{12}}\right)v_2 - \frac{1}{R_{12}}v_1 = \underline{\Omega}_2$$

For  $v_1=5, v_2=-10$

$$\underline{D}_1 = 1, \underline{D}_2 = -1$$

$$5\left(\frac{1}{R_1} + \frac{1}{R_{12}}\right) + \frac{10}{R_{12}} = 1 \Rightarrow \frac{5}{R_1} + \frac{15}{R_{12}} = 1$$

$$-10\left(\frac{1}{R_2} + \frac{1}{R_{12}}\right) - \frac{5}{R_{12}} = -1 \Rightarrow \frac{10}{R_2} + \frac{15}{R_{12}} = 1$$

Here  $R_{12} > 15$  since  $\frac{15}{R_{12}} < 1$  to have  $R_2 > 0$

$$\text{let } R_{12} = 30 \Rightarrow \frac{5}{R_1} + \frac{15}{30} = 1 \Rightarrow R_1 = 10$$

$$\Rightarrow \frac{10}{R_2} + \frac{15}{30} = 1 \Rightarrow R_2 = 20$$

i.e.  $R_1 = 10, R_2 = 20$  and  $R_{12} = 30$

#5

$$S^H(h_g) = \frac{2}{1+e^{-10h_g}} - 1$$

$$S^g(g_r) = \frac{2}{1+e^{-100g_r}} - 1$$

$$S^y(y_j) = y_j$$

$$a_1, g_1 = 10 \Rightarrow S^g(g_1) = S^g(10) \approx 1$$

$$g_2 = -20 \Rightarrow S^g(g_2) = S^g(-20) \approx -1$$

$$h_1 = S^g(g_1) w_{1,1}^{g-H} + S^g(g_2) w_{2,1}^{g-H} = 1 \cdot 2 + (-1)(-5) = 7$$

$$h_2 = S^g(g_1) w_{1,2}^{g-H} + S^g(g_2) w_{2,2}^{g-H} = 1(-2) + (-1)(-1) = -1$$

$$h_3 = S^g(g_1) w_{1,3}^{g-H} + S^g(g_2) w_{2,3}^{g-H} = 1(-4) + (-1)5 = -9$$

$$\Rightarrow S^H(h_1) = S^H(7) \approx 1$$

$$S^H(h_2) = S^H(-1) \approx -1$$

$$S^H(h_3) = S^H(-9) \approx -1$$

$$y_1 = S^H(h_1) w_{1,1}^{H-Y} + S^H(h_2) w_{2,1}^{H-Y} + S^H(h_3) w_{3,1}^{H-Y}$$

$$= 1 \cdot 3 + (-1)(-1) + (-1)(-4) = 8$$

$$y_2 = S^H(h_1) w_{1,2}^{H-Y} + S^H(h_2) w_{2,2}^{H-Y} + S^H(h_3) w_{3,2}^{H-Y}$$

$$= 1(-2) + (-1)(-1) + (-1)(-1) = 0$$

$$\Rightarrow S^y(y_1) = S^y(8) = 8$$

$$S^y(y_2) = S^y(0) = 0$$

b,,  $\Delta m_{q,j}^{H-Y} = [d_j - S^y(y_j)] S'_j(y_j) S''(h_q)$

$$= [d_j - y_j] S''(h_q) \text{ since } S^y(y_j) = y_j$$

and  $S^y(y_j) = 1$

$$\text{so } \Delta m_{1,1}^{H-Y} = [d_1 - y_1] S''(h_1) = [8 - 8] 1 = 0$$

$$\Rightarrow m_{1,1}^{H-Y}(+) = m_{1,1}^{H-Y} + \Delta m_{1,1}^{H-Y} = 3 + 0 = 3$$

$$\Delta m_{1,2}^{H-Y} = [d_2 - y_2] S''(h_1) = [5 - 0] 1 = 5$$

$$\Rightarrow m_{1,2}^{H-Y}(+) = m_{1,2}^{H-Y} + \Delta m_{1,2}^{H-Y} = -2 + 5 = 3$$

c,,  $\Delta m_{i,q}^{Q-H} = - \left[ \sum_{j=1}^2 [d_j - S^y(y_j)] S'_j(y_j) m_{q,j}^{H-Y}(+) \right]$

$$\cdot S''(h_q) S^q(g_i)$$

$$= - \left[ \sum_{j=1}^2 [d_j - y_j] m_{q,j}^{H-Y}(+) \right] \frac{10}{2} (1 - S''(h_q)) S^q(g_i)$$

$$\Delta m_{1,1}^{Q-H} = - \left[ \sum_{j=1}^2 [d_j - y_j] m_{1,j}^{H-Y}(+) \right] 5 (1 - S''(h_1)) S^q(g_1)$$

$$= - \left[ [8 - 8] 3 + [5 - 0] (-2) \right] 5 (1 - 1^2) 1$$

$$= 0 \Rightarrow m_{1,1}^{Q-H}(+) = m_{1,1}^{Q-H} + \Delta m_{1,1}^{Q-H} = 2 + 0 = 2$$