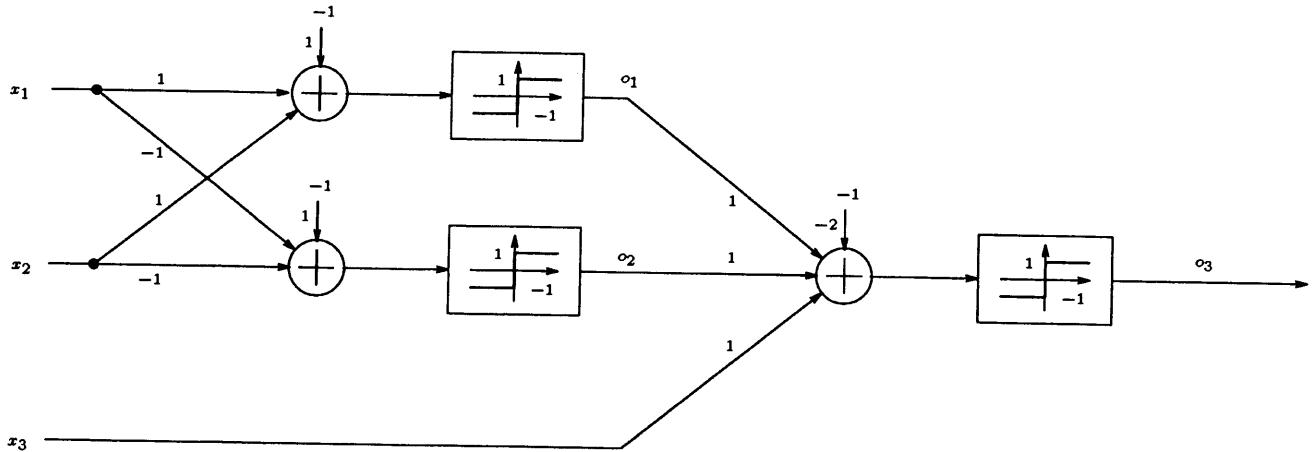


1. The network shown below implements a bipolar logic function of the inputs. Analyze the network, and determine the logic function in terms of x_1 , x_2 , and x_3 , assuming $x_i = \pm 1$, for $i = 1, 2, 3$. (20pts)



2. The decision surface of a single-layer perceptron classifier is given to be

$$g(x) = x_1 + x_2 + 1 = 0,$$

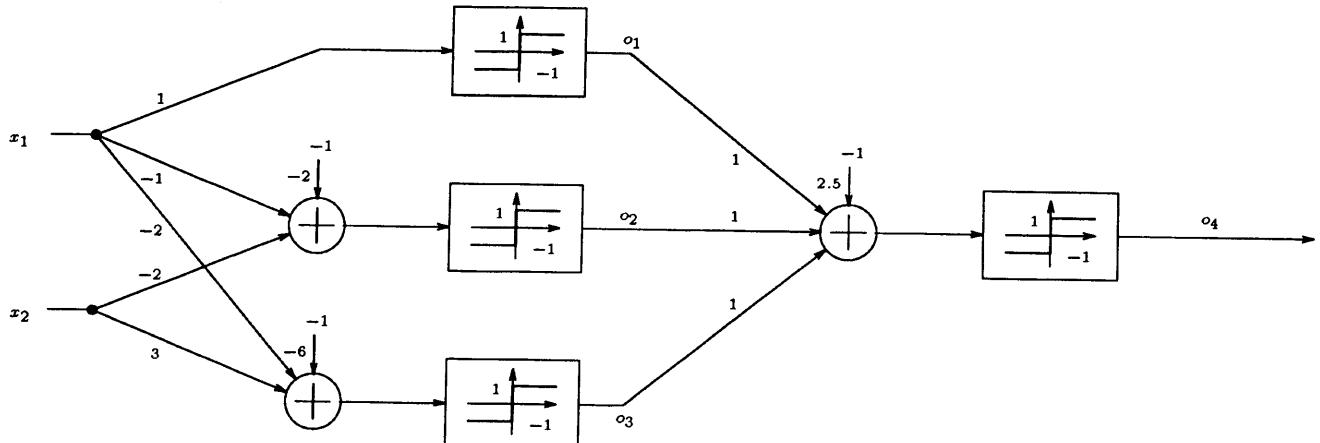
where $x = [x_1 \ x_2]^T$.

- (a) Determine two discriminant functions $g_1(x)$ and $g_2(x)$ by first finding two perpendicular vectors to the decision surface, and then by augmenting the vectors, such that $x = [0 \ 0]^T$ belongs to Class 1. (20pts)

- (b) Implement the classifier determined in part 2a by using

- i. a threshold logic unit, and (5pts)
- ii. a maximum selector. (5pts)

3. Given the network below, determine and plot the region for $o_4 = 1$ on the x_1 - x_2 plane. (25pts)



4. Given the two-layer network below, compute the updated values for the weights

(a) w_{11} and w_{21} ,

(10pts)

(b) v_{11} ,

(15pts)

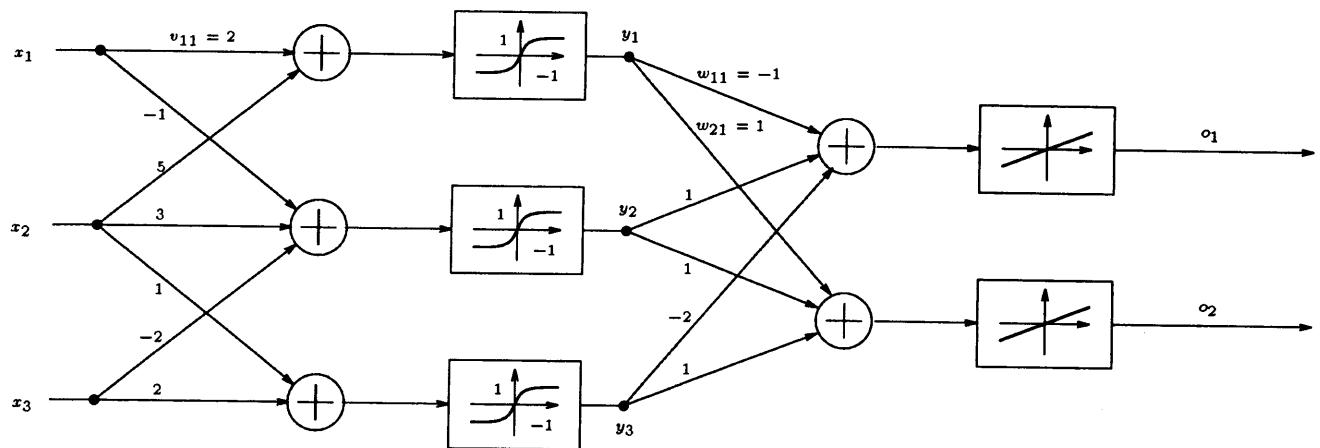
when the desired output signals are $d_1 = 2$, and $d_2 = -5$. Assume the activation functions generating y_i , for $i = 1, 2, 3$ are

$$f_{y_i}(\text{net}) = \frac{2}{1 + e^{-\text{net}}} - 1,$$

the activation functions generating o_i , for $i = 1, 2$ are

$$f_{o_i}(\text{net}) = \text{net},$$

and the gradient-descent step size $\eta = 1$.



$$\#1 \quad O_1 = \text{sgn}(x_1 + x_2 - 1)$$

x_1	x_2	$x_1 + x_2 - 1$	O_1
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$$\begin{array}{cccc} -1 & -1 & -3 & -1 \\ -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 \end{array} \} \Rightarrow O_1 = x_1 x_2$$

and

$$O_2 = \text{sgn}(-x_1 - x_2 - 1)$$

x_1	x_2	$-x_1 - x_2 - 1$	O_2
-1	-1	1	1
-1	1	-1	-1
1	-1	-1	-1
1	1	-3	-1

$$\} \Rightarrow O_2 = \overline{x}_1 \overline{x}_2$$

negation

$$O_3 = \text{sgn}(O_1 + O_2 + x_3 + 2)$$

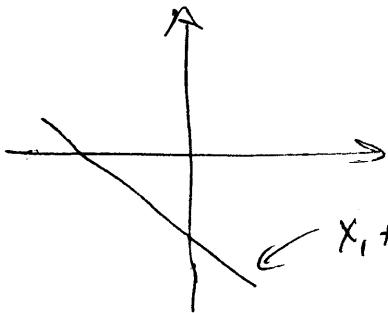
O_1	O_2	x_3	$O_1 + O_2 + x_3 + 2$	O_3
-1	-1	1	-1	-1
-1	-1	1	-1	-1
-1	1	1	-3	-1
1	-1	1	-1	1
1	1	1	3	1
			5	

$$O_3 = O_1 + O_2 + x_3$$

or

$$\text{So } O_3 = x_1 x_2 + \overline{x}_1 \overline{x}_2 + x_3$$

#2



$$x_1 + x_2 + 1 = 0 \Rightarrow [1 \ 1 \ 1] \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} = 0$$

$$\text{+ vector } = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\text{another + vector } = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

Let an augmented vector be $r_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ arbitrary

& another be

$$r_2 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

arbitrary but
not = -1

Then the augmented eqn.

$$[1 \ 1 \ 1] \begin{bmatrix} x_1 - a \\ x_2 - b \\ x_3 - c \end{bmatrix} = 0$$

$$[-1 \ -1 \ 1] \begin{bmatrix} x_1 - a \\ x_2 - b \\ x_3 - c \end{bmatrix} = 0$$

To find $a, b, \& c$, pick a point on the line $x_1 + x_2 + 1 = 0$
assuming $x_3 = 1$

$$\text{let } x_1 = 0 \ \& \ x_2 = -1 \Rightarrow$$

$$\begin{aligned} a &= 0 && \leftarrow \text{from } x_1 \\ b &= -1 && \leftarrow \text{from } x_2 \\ c &= 1 && \leftarrow \text{from } x_3 \end{aligned}$$

Then $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2+1 \\ x_3-1 \end{bmatrix} = 0$, $x_1 + x_2 + 1 + x_3 - 1 = 0$
 $x_1 + x_2 + x_3 = 0$

let $g_i = x_3$

$\Rightarrow g_i = -x_1 - x_2$

One of the discriminant functions

$\begin{bmatrix} -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2+1 \\ x_3-1 \end{bmatrix} = 0$, $-x_1 - x_2 - 1 + x_3 - 1 = 0$
 $-x_1 - x_2 + x_3 - 2 = 0$

let $g_j = x_3$

$\Rightarrow g_j = x_1 + x_2 + 2$

The other discriminant function

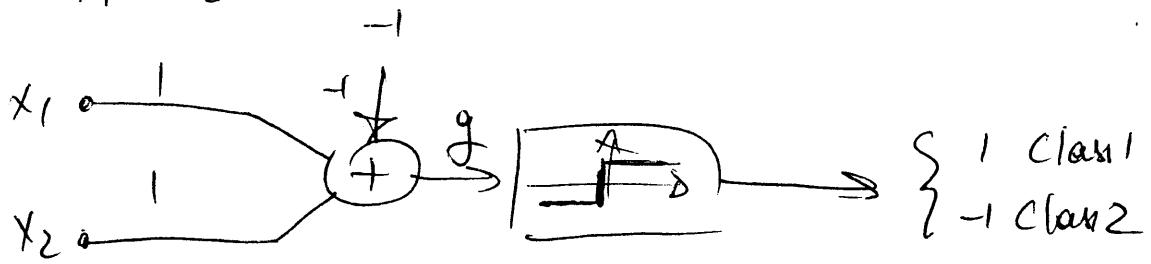
To decide on $i \& j$, let

$$x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ be in class 1} \implies g_i = 0 \\ g_j = 2$$

$$\text{so } j=1 \quad \text{or} \quad g_1 = x_1 + x_2 + 2 \\ g_2 = -x_1 - x_2$$

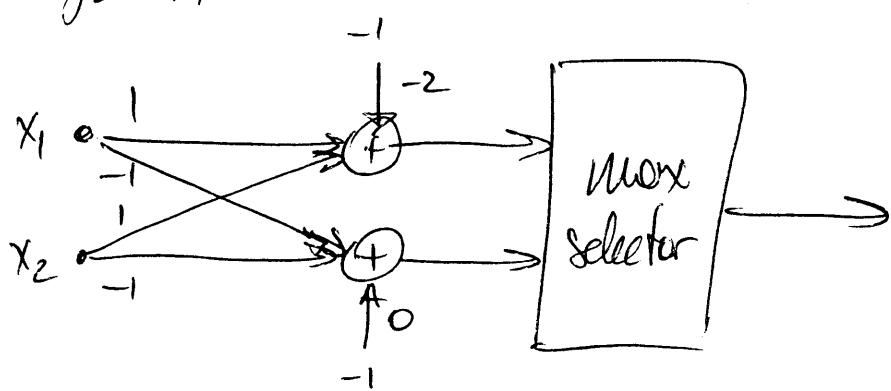
NOTE $g(x) = g_1(x) - g_2(x) = x_1 + x_2 + 2 - (-x_1 - x_2)$
 $= 2x_1 + 2x_2 + 2$

$$\textcircled{1} \quad g = x_1 + x_2 + 1$$



$$\textcircled{ii} \quad g_1 = x_1 + x_2 + 2$$

$$g_2 = -x_1 - x_2$$



$$\#3 \quad o_1 = \operatorname{sgn}(x_1)$$

$$o_2 = \operatorname{sgn}(-x_1 - 2x_2 + 2)$$

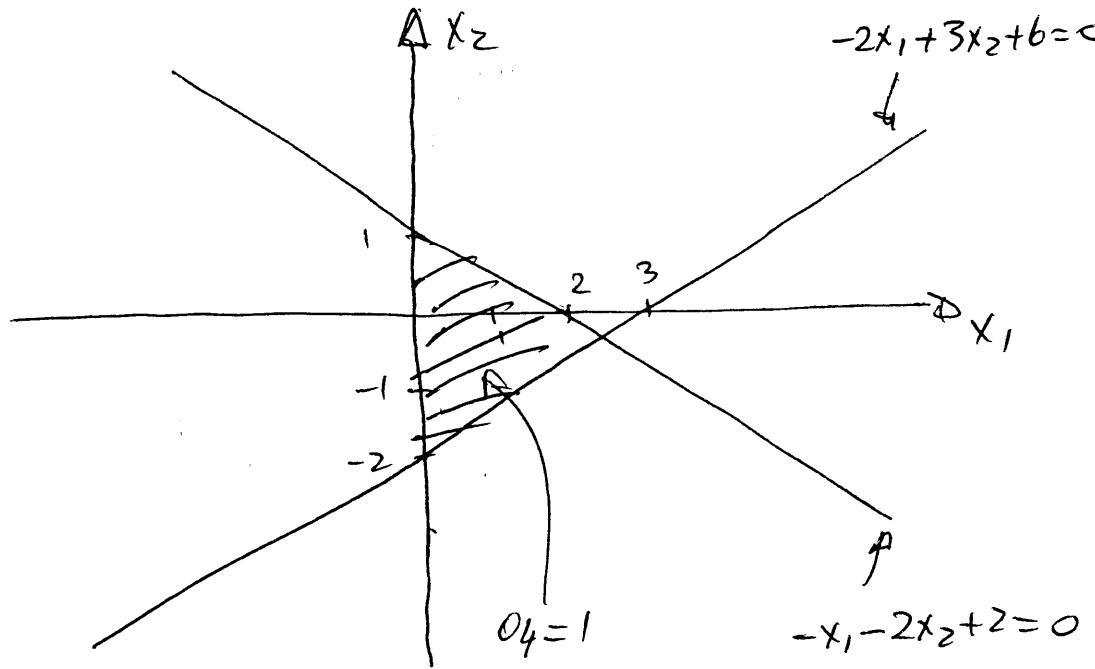
$$o_3 = \operatorname{sgn}(-2x_1 + 3x_2 + 6)$$

$$o_4 = \operatorname{sgn}(o_1 + o_2 + o_3 - 2.5)$$

$$o_1 \quad o_2 \quad o_3 \quad o_1 + o_2 + o_3 - 2.5 \quad o_4$$

-1	-1	-1	-5.5	-1
-1	-1	-1	-3.5	-1
-1	-1	-1	-3.5	-1
-1	-1	-1	-1.5	-1
-1	-1	-1	-3.5	-1
-1	-1	-1	-1.5	-1
-1	-1	-1	-1.5	-1
-1	-1	-1	0.5	2 } 22

$$o_4 = o_1 o_2 o_3$$



#4 $y_1 = f_{y_1}(2x_1 + 5x_2) = f_{y_1}(2) = 0.7616$
 $y_2 = f_{y_2}((-1)x_1 + 3x_2 + (-2)(-2)) = f_{y_2}(3) = 0.9051$
 $y_3 = f_{y_3}(1x_1 + 2(-2)) = f_{y_3}(-4) = -0.9640$

$$o_1 = (-1)(0.7616) + (1)(0.9051) + (-2)(-0.9820) = 2.0716$$

$$o_2 = (1)(0.7616) + (1)(0.9051) + (1)(-0.9820) = 0.7027$$

$$\delta o_1 = (d_1 - o_1) \vec{f}_{o_1} = (d_1 - o_1) = 2 - 2.0716 = -0.0716$$

$$\delta o_2 = (d_2 - o_2) \vec{f}_{o_2} = (d_2 - o_2) = 5 - 0.7027 = 5.7027$$

θ_1 $w_{11}(+) = w_{11} + \eta \delta o_1 y_1$
 $= -1 + (1)(-0.0716)(0.7616) = -1.0545$

$w_{21}(+) = w_{21} + \eta \delta o_2 y_1$
 $= + + (1)(-5.7027)(0.7616) = -3.3431$

$$\text{b)} \quad \delta y_j = f'_j \sum_{k=1}^2 \delta o_k w_{kj} \\ = \frac{1}{2} (1-y_j^2) (\delta o_1 w_{1j} + \delta o_2 w_{2j}) \quad \text{since } f'_j = \frac{1}{2}(1-y_j^2)$$

$$\begin{aligned} \delta y_1 &= \frac{1}{2} (1-y_1^2) (\delta o_1 w_{11} + \delta o_2 w_{21}) \\ &= \frac{1}{2} (1-(0.7616)^2) ((-0.0716)(-1) + (-5.7027)(1)) \\ &= -1.1825 \end{aligned}$$

$$\begin{aligned} v_{11}(+) &= v_{11} + \eta \delta y_1 x_1 \\ &= 2 + (1)(-1.1825)(1) = 0.8175 \end{aligned}$$