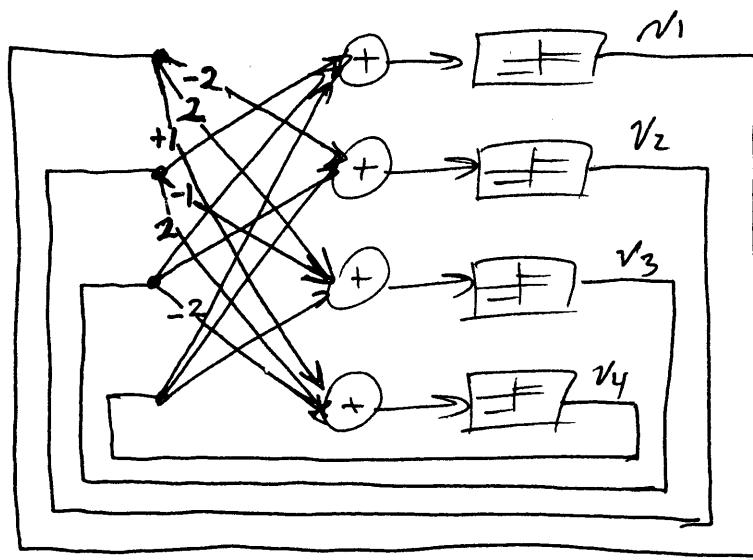


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1. Consider the following discrete-time recurrent network with bipolar neurons.



- (a) Determine the weight matrix  $W$  from the figure by inspection. (5pts)
- (b) Analyze the synchronous updates and the corresponding energy value updates starting from the initial states  $[1, 1, -1, 1]^T$  and  $[1, 1, 1, 1]^T$ . (10pts)
- (c) Analyze the asynchronous updates and the corresponding energy value updates starting from the same initial states  $[1, 1, -1, 1]^T$  and  $[1, 1, 1, 1]^T$ . Assume the updates take place in natural order starting with the first neuron. (10pts)

2. The dynamical equations of a linear system are given by

$$\begin{aligned}\dot{x}_1 &= -2x_1 + x_2, \\ \dot{x}_2 &= 5x_1 - 3x_2.\end{aligned}$$

- (a) Find a Lyapunov function for this system. (+15pts)

The dynamical equations of another system are given by

$$\begin{aligned}\dot{x}_1 &= -\sin(2x_1) + \tan(x_2), \\ \dot{x}_2 &= \frac{5x_1}{1+x_1^2} + e^{-3x_2} - 1.\end{aligned}$$

- (b) Find an equilibrium state of this nonlinear system, and check the local stability about this equilibrium state using Lyapunov's first method. (+10pts)

3. A continuous-time control plant has two unknown parameters  $\alpha$  and  $\beta$  in its dynamics given by

$$\ddot{y} + \frac{\alpha}{\beta}\dot{y} = 2\beta u.$$

An identifier is to be designed for this plant to identify the unknown parameters  $\alpha$  and  $\beta$ . The identifier minimizes an incremental error cost function which is given by

$$J = \frac{1}{2} \frac{d}{dt} (y - \hat{y})^2,$$

where  $y$  is the output of the plant, and  $\hat{y}$  is the output of the identifier.

- (a) Obtain sensitivity models of the plant with respect to  $\alpha$  and  $\beta$ . (10pts)
- (b) Obtain update equations for  $\alpha$  and  $\beta$  that iteratively minimizes the error cost function  $J$ . (5pts)
- (c) Draw the block diagram showing the connections among the plant, the identifier, and the sensitivity models. Clearly mark all the inputs and outputs of the blocks, and show the signals needed for the training of the networks. (10pts)

4. A feedforward single layer associative memory network with bipolar threshold activation functions is to be designed to store three pairs of vectors:

$$[1, -1]^T \longrightarrow [-1, 1, -1]^T,$$

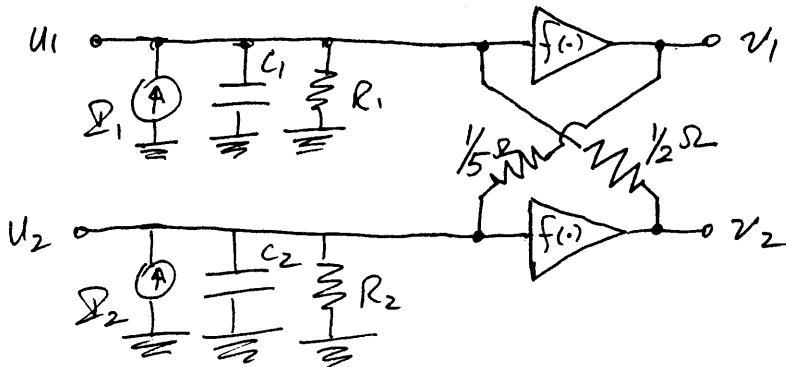
$$[-1, 1]^T \longrightarrow [1, -1, 1]^T,$$

and

$$[1, 1]^T \longrightarrow [1, 1, 1]^T.$$

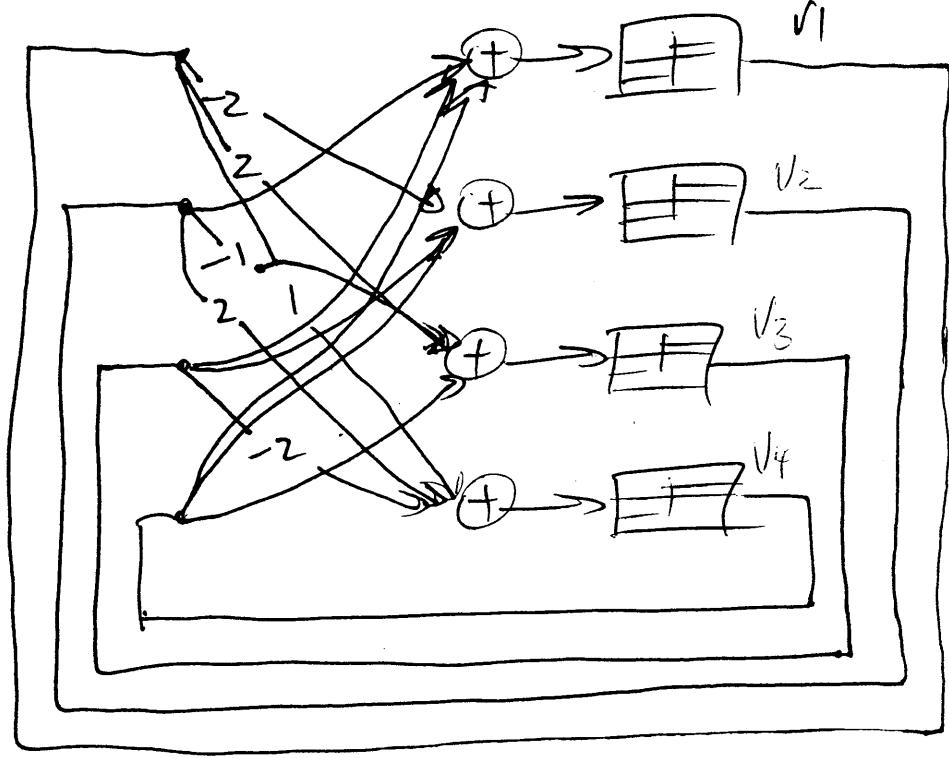
- (a) Determine the weight matrix  $W_1$  based on the Hebbian learning. (10pts)
- (b) Determine the weight matrix  $W_2$  based on the minimal error analysis. (10pts)
- (c) Find, if possible, a bipolar input, such that there is a difference at the output of the two methods. Explain your findings. (5pts)

5. Consider the following continuous-time single layer feedback network.



- (a) Obtain the state equations. (10pts)
- (b) Determine the weight matrix  $W$ . (5pts)
- (c) Determine the equilibrium states for high-gain neurons and for  $I_1 = I_2 = 0$ . (10pts)

#1

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$$Q_{11} \quad w_{12} = w_{21} = -2$$

$$w_{13} = w_{31} = 2$$

$$w_{14} = w_{41} = 1$$

$$w_{23} = w_{32} = -1$$

$$w_{24} = w_{42} = 2$$

$$w_{34} = w_{43} = -2$$

$$\Rightarrow W = \begin{bmatrix} 0 & -2 & 2 & 1 \\ -2 & 0 & -1 & 2 \\ 2 & -1 & 0 & -2 \\ 1 & 2 & -2 & 0 \end{bmatrix}$$

## b) Synchronous Updates

$\underline{v^k}$

$\underline{w_{vk}}$

$\underline{P[w_{vk}]}$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -2 & 2 & 1 \\ -2 & 0 & -1 & \frac{1}{2} \\ 2 & -1 & 0 & -\frac{1}{2} \\ 1 & 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -5 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}^4$$

$$\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -2 & 2 & 1 \\ -2 & 0 & -1 & \frac{1}{2} \\ 2 & -1 & 0 & -\frac{1}{2} \\ 1 & 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \\ -5 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

In other words,

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}}$$

$\underline{v^k}$

$\underline{w_{vk}}$

$\underline{P[w_{vk}]}$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -2 & 2 & 1 \\ -2 & 0 & -1 & \frac{1}{2} \\ 2 & -1 & 0 & -\frac{1}{2} \\ 1 & 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -2 & 2 & 1 \\ -2 & 0 & -1 & \frac{1}{2} \\ 2 & -1 & 0 & -\frac{1}{2} \\ 1 & 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

In other words,

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

## c) Asynchronous Updates in natural order

$$\frac{v^k}{\equiv} \quad \underline{\underline{W_i v^k}} \quad \underline{\underline{T(W_i v^k)}}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & -2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\xrightarrow{\text{G}} \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} -2 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -5 \\ 1 \end{pmatrix}$$

$$\xrightarrow{\text{→}} \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} \cdot & & & \\ 1 & 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 3 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\textcircled{1} \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & -2 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$\underline{v^k}$ 

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

 $\underline{W^k}$ 

$$\begin{bmatrix} 0 & -2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

 $P[W, v^k]$ 

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

#2

$$\begin{aligned}\dot{x}_1 &= -2x_1 + x_2 \\ \dot{x}_2 &= 5x_1 - 3x_2\end{aligned} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Q11 System is linear, so

$L(x) = x^T P x$  is a Lyapunov function

for  $A^T P + PA = -Q$

where  $P > 0$  and  $Q \geq 0$

To find  $P$ , let  $Q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

and let  $P = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \leftarrow \text{symm.}$

$$\begin{bmatrix} -2 & 5 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} + \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

1,1 term  $-2P_1 + 5P_2 - 2P_1 + 5P_2 = -1$   
 $-4P_1 + 10P_2 = -1$

1,2 term  $-2P_2 + 5P_3 + P_1 - 3P_2 = 0$   
 $P_1 - 5P_2 + 5P_3 = 0$

2,2 term  $P_2 - 3P_3 + P_2 - 3P_3 = -1$   
 $2P_2 - 6P_3 = -1$

13-099  
 42-381 500 SHEETS FILLED 5 SQUARE  
 42-382 500 SHEETS IN EACH 5 SQUARE  
 42-383 100 SHEETS IN EACH 5 SQUARE  
 42-384 200 SHEETS IN EACH 5 SQUARE  
 42-392 100 HIG-CYC 500 SHEETS  
 42-393 200 HIG-CYC 500 SHEETS  
 42-399 200 HIG-CYC 500 SHEETS

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1,1 term

$$P_1 = \frac{10P_2 + 1}{4}$$

2,2 term

$$P_3 = \frac{2P_2 + 1}{6}$$

1,2 term

$$\frac{10P_2 + 1}{4} - 5P_2 + 5 \frac{2P_2 + 1}{6} = 0$$

$$3(10P_2 + 1) - 60P_2 + 10(2P_2 + 1) = 0$$

$$-10P_2 + 13 = 0$$

$$P_2 = \frac{13}{10} \Rightarrow P_1 = \frac{7}{2}$$

$$P_3 = \frac{3}{5}$$

$$\Rightarrow P = \begin{bmatrix} 7/2 & 13/10 \\ 13/10 & 3/5 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 35 & 13 \\ 13 & 6 \end{bmatrix}$$

$$\text{so } L(x) = \frac{1}{10} [x_1 x_2] \begin{bmatrix} 35 & 13 \\ 13 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \frac{1}{10} [35x_1 + 13x_2 \quad 13x_1 + 6x_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \frac{1}{10} (35x_1^2 + 13x_1x_2 + 13x_1x_2 + 6x_2^2)$$

$$= \frac{1}{10} (35x_1^2 + 26x_1x_2 + 6x_2^2)$$

$$L(x_1, x_2) = 3.5x_1^2 + 2.6x_1x_2 + 0.6x_2^2$$

$$\dot{x}_1 = -\sin(2x_1) + \tan(x_2)$$

$$\dot{x}_2 = \frac{5x_1}{1+x_1^2} + e^{-3x_2} - 1$$

 $b_{11}$ 

Equilibrium

state

 $\Rightarrow$ 

$$\begin{aligned}\dot{x}_1 &= 0 & -\sin(2x_1) + \tan(x_2) &= 0 \\ \dot{x}_2 &= 0 & \frac{5x_1}{1+x_1^2} + e^{-3x_2} - 1 &= 0\end{aligned}$$

By inspection, we can see that  
 $x_1 = x_2 = 0$  is an equilibrium pt.

Lyapunov's first method  $\rightarrow$  linear approx.

$$\begin{aligned}\text{let } \dot{x}_1 &= f_1(x_1, x_2) \\ \dot{x}_2 &= f_2(x_1, x_2)\end{aligned}$$

$$\left. \frac{\partial f_1}{\partial x_1} \right|_{x_1=x_2=0} = -2 \cos(2x_1) \Big|_{x_1=x_2=0} = -2$$

$$\left. \frac{\partial f_1}{\partial x_2} \right|_{x_1=x_2=0} = \left( 1 + \tan^2(x_2) \right) \Big|_{x_1=x_2=0} = 1$$

$$\left. \frac{\partial f_2}{\partial x_1} \right|_{x_1=x_2=0} = \frac{5(1+x_1^2) - 5x_1(2x_1)}{(1+x_1^2)^2} \Big|_{x_1=x_2=0} = 5$$

$$\left. \frac{\partial f_2}{\partial x_2} \right|_{x_1=x_2=0} = -3 e^{-3x_2} \Big|_{x_1=x_2=0} = -3$$

linear  
Approx

$$\begin{aligned}\dot{x}_1 &= -2x_1 + x_2 \\ \dot{x}_2 &= 5x_1 - 3x_2\end{aligned}$$

A

Same system as in part a,  
since there was a Lyapunov  
func for the system, then  
the nonlinear system is also  
LOCALLY asymptotically stable  
about  $x_1 = x_2 = 0$ .

#3

$$\ddot{y} + \frac{\alpha}{\beta} \dot{y} = 2\beta \ddot{u}$$

$$\text{Cost } J = \frac{1}{2} \frac{d}{dt} (y - \hat{y})^2$$

a) Sensitivity models give  $y_\alpha$  and  $y_\beta$

$$\frac{\partial}{\partial \alpha} (\ddot{y} + \frac{\alpha}{\beta} \dot{y} = 2\beta \ddot{u})$$

$$\ddot{y}_\alpha + \frac{1}{\beta} \dot{y} + \frac{\alpha}{\beta} \dot{y}_\alpha = 0$$

$$\ddot{y}_\alpha + \frac{\alpha}{\beta} \dot{y}_\alpha = -\frac{1}{\beta} \dot{y}$$

$$\frac{\partial}{\partial \beta} (\ddot{y} + \frac{\alpha}{\beta} \dot{y} = 2\beta \ddot{u})$$

$$\ddot{y}_\beta - \frac{\alpha}{\beta^2} \dot{y} + \frac{\alpha}{\beta} \dot{y}_\beta = 2\ddot{u}$$

$$\ddot{y}_\beta + \frac{1}{\beta} \dot{y}_\beta = 2\ddot{u} + \frac{\alpha}{\beta^2} \dot{y}$$

b) Update equations

$$\alpha(+) = \alpha - \gamma \frac{dJ}{d\alpha}$$

$$\beta(+) = \beta - \gamma \frac{dJ}{d\beta}$$

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42-383 100 SHEETS EYE-EASE® 5.5 SQUARE  
42-384 100 HEAVY EASY-SQUARE 5.5 SQUARE  
42-385 200 RECYCLED WHITE 5.5 SQUARE  
42-386 200 RECYCLED WHITE 5.5 SQUARE  
Map-Tite® A

①

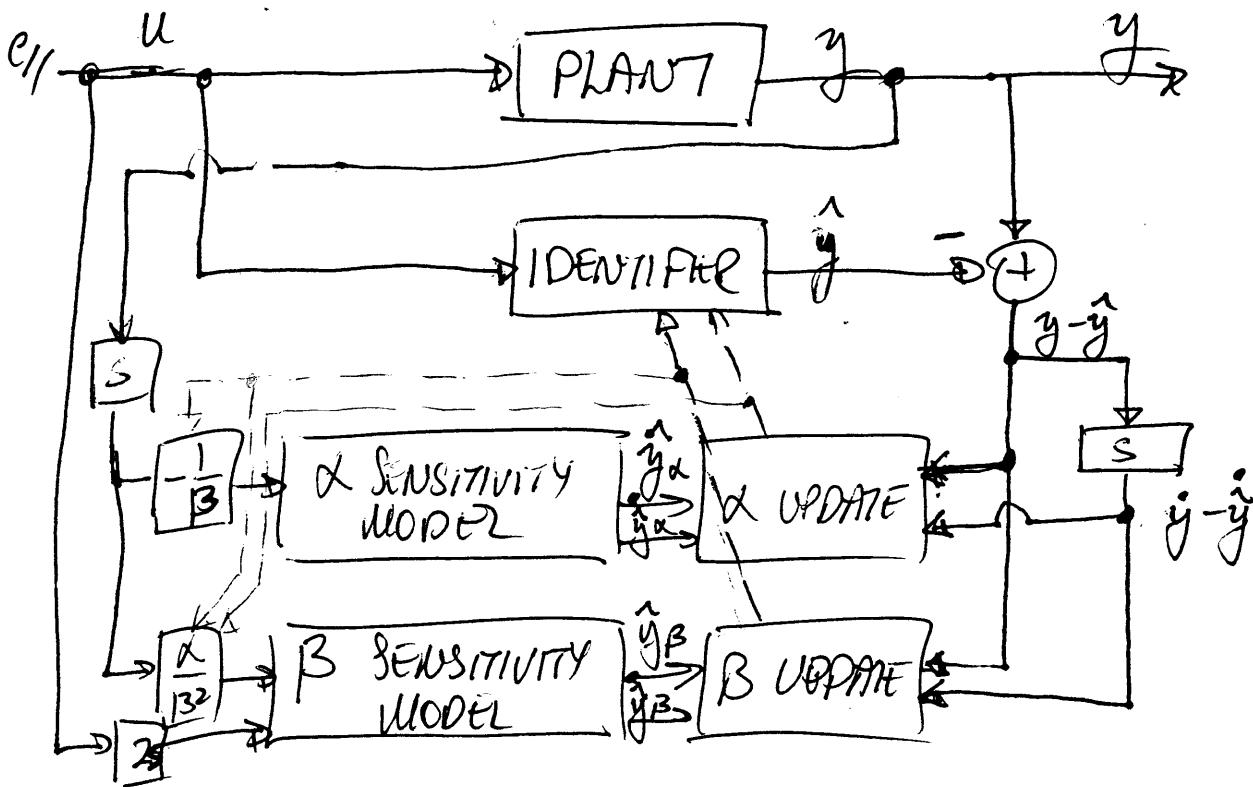
②

$$\frac{d\bar{\tau}}{d\alpha} = \frac{1}{2} \frac{d}{dt} \left( \frac{d}{d\alpha} (y - \hat{y})^2 \right)$$

$$= \frac{1}{2} \frac{d}{dt} ((y - \hat{y})(-\hat{y}'_\alpha))$$

$$= -(y - \hat{y}) \hat{y}'_\alpha - (y - \hat{y}) \hat{y}'_\alpha$$

$$\frac{d\bar{\tau}}{d\beta} = - (y - \hat{y}) \hat{y}'_\beta - (y - \hat{y}) \hat{y}'_\beta \quad \text{similarly}$$



#4

$$s^{(1)} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \rightarrow \begin{bmatrix} -1 \\ -1 \end{bmatrix} = f^{(1)}$$

$$s^{(2)} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix} = f^{(2)}$$

$$s^{(3)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix} = f^{(3)}$$

a, Hebbian weight matrix  $W_1 = FS^T$

$$F = [f^{(1)} \ f^{(2)} \ f^{(3)}] = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$S = [s^{(1)} \ s^{(2)} \ s^{(3)}] = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 3 & -1 \\ -1 & 3 \end{bmatrix}$$

b, Minimal error weight matrix  $W_2 = FS^*$

$$S^* = S^T (S S^T)^{-1} \quad \text{pseudo-inv.}$$

since  $S S^T$  is invertible

$$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} \left( \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} \right)^{-1}$$

$$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}^{-1}$$

$$S^* = \frac{1}{3 \times 3 - (-1)(-1)} \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} 2 & -2 \\ -2 & 2 \\ 4 & 4 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 2 & 2 \end{bmatrix}$$

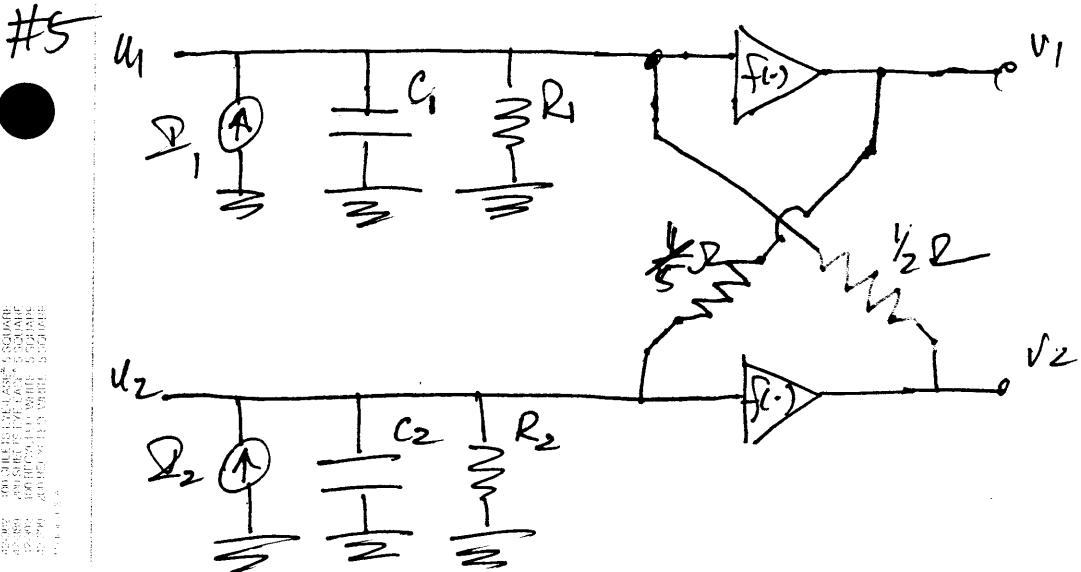
$$W_2 = \frac{1}{4} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 0 & 4 \\ 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

c) The difference between  $W_1$  and  $W_2$

$\underline{x_1}$	$\underline{x_2}$	$(W_1x), (W_1x)_2$	$(W_2x), (W_2x)_3$	$(W_1x), (W_2x)_2$	$(W_2x), (W_2x)_3$
-1	-1	-2	-2	-1	-1
-1	-1	-4	-4	-1	-1
1	-1	2	2	1	1

After activation function  
both give the same output.



a) Node eqns

$$-\mathfrak{D}_1 + C_1 \frac{du_1}{dt} + \frac{1}{R_1} u_1 + \frac{1}{R_2} (u_1 - v_2) = 0$$

$$-\mathfrak{D}_2 + C_2 \frac{du_2}{dt} + \frac{1}{R_2} u_2 + \frac{1}{R_1} (u_2 - v_1) =$$

or  $C_1 \frac{du_1}{dt} = 2v_2 - \left(2 + \frac{1}{R_1}\right)u_1 + \mathfrak{D}_1$

$$C_2 \frac{du_2}{dt} = 5v_1 - \left(5 + \frac{1}{R_2}\right)u_2 + \mathfrak{D}_2$$

b) From the state eqns.

$$\omega = \begin{bmatrix} 0 & 2 \\ 5 & 0 \end{bmatrix}$$

$e_{11}$  High gain  $\Rightarrow v_1 \approx 0, v_2 \approx 0$

$$E = -\frac{1}{2} v^T W v - c^T v$$

$$\Omega_1 = \Omega_2 = 0 \Rightarrow E = -\frac{1}{2} v^T W v$$

$$E = -\frac{1}{2} [v_1 \ v_2] \begin{bmatrix} 0 & 2 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= -\frac{1}{2} [5v_2 \ 2v_1] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= -\frac{1}{2} (5v_1 v_2 + 2v_1 v_2) = -\frac{7}{2} v_1 v_2$$

Equilibrium states when  $E$  is minimum

$$\begin{array}{ccccc} v_1 & & v_2 & & \equiv E \\ \hline -1 & & -1 & & -7/2 \\ -1 & & 1 & & 7/2 \\ 1 & & -1 & & 7/2 \\ & & 1 & & -7/2 \end{array} \quad \begin{array}{c} \nearrow \\ \text{equilibrium states} \end{array}$$

So equilibrium states are  $\begin{bmatrix} -1 \\ -1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ 1 \end{bmatrix}$