Computer Engineering 111
Test 1
September 26, 2002

Name SOLUTIONS

Eleven problems, 100 points.

Closed books, closed notes, no calculators. You would be wise to read all problems before beginning, note point values and difficulty of problems, and budget your time accordingly.

Please do not open the test until I tell you to do so.

Good luck!

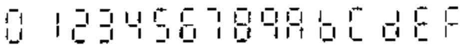
1. (2 points) Consider the statement A + (AB) = A

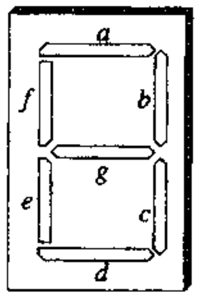
Is this true in ordinary Algebra? Circle your answer. F

For example, consider A= 2 and B = 5.

Is this true in Boolean Algebra? Circle your answer. T

2. (10 points) Consider the 7-segment display in this figure. Imagine that the numbers 0 – 9 and the hexadecimal digits A-F are displayed as:





(For example, the number "1" is displayed by segments b and c in the figure.) With this information, create the truth table, Karnaugh map (on the next page), and minimal SOP expression for the logic to drive the LED labeled f, in the figure.

Hex	ABCD	f
0	0000	1
1	0001	0
2	0010	0
3	0011	0
4	0100	1
5	0101	1
6	0110	1
7	0111	0
8	1000	1
9	1001	1
A	1010	1
B	1011	1
C	1100	1
D	1101	0
E	1110	1
F	1111	1

2. (Continued)

AB \ CD	00	01	11	10
00	1	0	0	0
01	1	1	0	1
11	1	0	1	1
10	1	1	1	1

F = $\overline{C}\,\overline{D} + A\overline{B} + AC + \overline{A}B\overline{C} +$ [redacted] $B\overline{D}$

3. (11 total points)

a) (2 points) convert to hex and binary:

451371 (octal) = 100 101 001 011 111 001(binary) = 252F9 (hex)
= 0010 0101 0010 1111 1001

b) (2 points) convert to decimal:

10010011011011 (binary)= 9435 (decimal)

3 + 8 + 16 + 64 + 128 + 1024 + 8192 = 9216 + 192 + 27 = 9435

c) (5 points) convert to binary, octal and hex:

986.6875 (decimal) = 512 + 256 + 128 + 64 + 16 + 8 + 2 + .5 + .125 + 0.0625
= 1111011010.1011 (binary)
= 001 111 011 010.101 100 (binary)
= 0011 1101 1010.1011 (binary)

986.6875 (decimal) = 1732.54(octal)

986.6875 (decimal) = 3DA.B (hex)

d) (2 points) convert to octal and hex

1100101010111.000100101(binary) = 14527.045 (octal) = 1957.128 (hex)

001 100 101 010 111.000 100 101(binary)

0001 1001 0101 0111.0001 0010 1000 (binary)

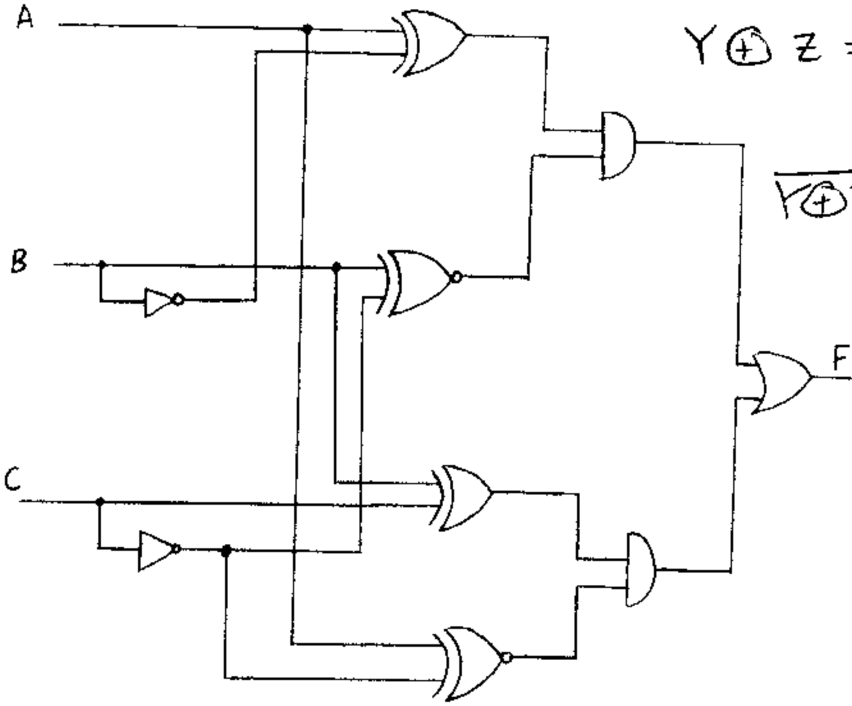
4). (18 points)

Recall that

$Y \oplus Z = \bar{Y}Z + Y\bar{Z}$

and

$\overline{Y \oplus Z} = \bar{Y}\bar{Z} + YZ$



a) Obtain the minimal SOP form for the function "F" using Boolean algebra.

$$F = (A \oplus \bar{B})(\overline{B \oplus \bar{C}}) + (B \oplus C)(\overline{A \oplus \bar{C}}) = (\bar{A}\bar{B} + AB)(\bar{B}C + B\bar{C}) + (\bar{B}C + B\bar{C})(\bar{A}C + A\bar{C})$$

$$= \bar{A}\bar{B}C + AB\bar{C} + \bar{A}\bar{B}C + AB\bar{C}$$

$$\therefore F = \bar{A}\bar{B}C + AB\bar{C}$$

b) Using a K-map and truth table obtain the minimal SOP. (Note that your answer should match the answer in "(a)." You can do this part first if you prefer.)

ABC	F
000	0
001	1
010	0
011	0
100	0
101	0
110	1
111	0

A \ BC	00	01	11	10
11	0	1	0	0
10	0	0	0	1

$F = \bar{A}\bar{B}C + AB\bar{C}$

c) Write the function in NAND-NAND form.

$F = \overline{\overline{\bar{A}\bar{B}C} \cdot \overline{AB\bar{C}}}$

5. (14 points) The Prime numbers between 0 and 9 are: 1, 2, 3, 5, 7. **Your input is in XS3.** F = 1 if the number is Prime, 0 if not. You must use don't cares for the invalid inputs, i.e., any input that does not code a number between 0 and 9. So, use this specification to determine whether the function F is a 1, 0 or X. Fill in the columns # and f in the truth table first. A few example values of f are already in the truth table to help you, just fill in the rest.
After you have filled in the whole truth table, write F in terms of minterms, and again in maxterms, i.e.:

$F = \Sigma m(4,5,6,8,10) + \Sigma mX(0,1,2,13,14,15\ )$

$F = \prod M(3,7,9,11,12)\prod MX(0,1,2,13,14,15)$

Finally, using a K-map, find the minimal SOP expression for F.

#	ABCD	F	Position
	0000	X	0
	0001	X	1
	0010	X	2
0	0011	0	3
1	0100	1	4
2	0101	1	5
3	0110	1	6
4	0111	0	7
5	1000	1	8
6	1001	0	9
7	1010	1	10
8	1011	0	11
9	1100	0	12
	1101	X	13
	1110	X	14
	1111	X	15

AB \ CD	00	01	11	10
00	X	X	0	X
01	1	1	0	1
11	0	X	X	X
10	1	0	0	1

$F = \overline{B}\,\overline{D} + \overline{A}\,\overline{C} + \overline{A}\,\overline{D} = \overline{B}\,\overline{D} + \overline{A}\,\overline{C} + C\overline{D}$ either is OK

6. (9 points)

$F = \prod M(2,3,6,8,11,13,15)$

Write the <u>canonical</u> **SOP** expression and the <u>minimal</u> **POS** expression for F.

Decimal	ABCD	F
0	0000	1
1	0001	1
2	0010	0
3	0011	0
4	0100	1
5	0101	1
6	0110	0
7	0111	1
8	1000	0
9	1001	1
10	1010	1
11	1011	0
12	1100	1
13	1101	0
14	1110	1
15	1111	0

AB \ CD	00	01	11	10
00	1	1	0	0
01	1	1	1	0
11	1	0	0	1
10	0	1	0	1

Canonical SOP

$F = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}B\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}BC\overline{D} + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}CD + AB\overline{C}D + ABCD$

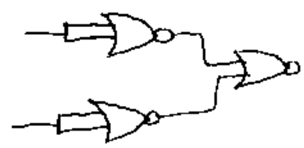
Minimal POS

$F = (\overline{A}+B+C+D)(\overline{A}+\overline{B}+\overline{D})(A+\overline{C}+D)(B+\overline{C}+\overline{D})$

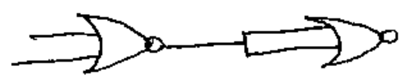
7. (3 points)

Show that you can implement any circuit with only NOR gates. To do this, just redraw the AND, OR, and INVERTER (i.e., NOT) gates, in NOR logic.

AND



OR



INVERTER

8. (9 points) F is given by the truth table below. Express F in minimal SOP form, and as a sum of minterms, i.e. $F = \Sigma m(0,1,5,7,8,9,10,12,15)$.

SOP Form: F =

Decimal	ABCD	F
0	0000	1
1	0001	1
2	0010	0
3	0011	0
4	0100	0
5	0101	1
6	0110	0
7	0111	1
8	1000	1
9	1001	1
10	1010	1
11	1011	0
12	1100	1
13	1101	0
14	1110	0
15	1111	1

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1		0
11	1	0		0
10		1	0	

$F = A\bar{B}\bar{D} + \bar{B}\bar{C} + A\bar{C}\bar{D} + BCD + \bar{A}\bar{C}D$

or

$F = A\bar{B}\bar{D} + \bar{B}\bar{C} + A\bar{C}\bar{D} + BCD + \bar{A}BD$

9. (11 points) F is given by the truth table below. Express F in minimal SOP form, and as a sum of minterms, i.e. $F = \Sigma m(1,3,5,6,11,12,15) + \Sigma mx(0,7,8,14)$.

SOP Form: F =

Decimal	ABCD	F
0	0000	x
1	0001	1
2	0010	0
3	0011	1
4	0100	0
5	0101	1
6	0110	1
7	0111	x
8	1000	x
9	1001	0
10	1010	0
11	1011	1
12	1100	1
13	1101	0
14	1110	x
15	1111	1

AB \ CD	00	01	11	10
00	X	1	1	0
01	0	1	X	1
11	1	0	1	X
10	X	0	1	0

$F = \bar{A}D + BC + CD + AB\bar{D}$

or

$F = \bar{A}D + BC + CD + A\bar{C}\bar{D}$

10. (8 points) Perform the following arithmetic in 8-bit binary, 2's complement arithmetic. Show your work and check to make sure it is correct.

a) 74 – 127 = 01001010 - 01111111 = 01001010 + 10000001

01001010
10000001

11001011 which is the complement of 00110101 = 53
Therefore 74 – 127 = -53.

b) 88 – 11 = 01011000 – 00001011 = 01011000 + 11110101

01011000
11110101

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1 01001101 (ignore the high 1 bit)

01001101 = 77.

c) – 18 + (- 23) = - 00010010 + (- 00010111) = 11101110 + 11101001

11101110
11101001

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1 11010111 (ignore the high 1 bit)

11010111 is the complement of 00101001 = 41.
Therefore –18 + (-23) = -41.

d) 20 X 6 = 00010100 x 00000110

00010100
x 00000110

00101000
+ 01010000

01111000 = 120.

11. (5 points) Let's do a word problem. I am pleased to announce my candidacy as a write-in candidate for US Senator. I'm too late to get on the ballot, so to compete with Talent and Carnahan, I will have to make a REALLY great campaign promise. Here it is: "**If** I am elected this year, **then** I will use my big-shot-Senator influence to get the whole class a free ride on the space shuttle. I appreciate your vote."

For this problem, the variable E is the event that I am elected US Senator in this election. The variable S is the event that the whole class gets a free ride on the space shuttle. These variables take on the value 0 if they are false, and 1 if they are true. They are only assigned after the election, and after I have had sufficient time to keep my promise if I intend to. The function F is 0 <u>IF YOU CAN PROVE</u> I was lying with the above promise, based on the events E and S, and 1 otherwise.

Carefully, fill in the truth table below and give the minimal SOP expression for F. (You probably won't need a K-map.) (The correct SOP form of F is worth 1 point. The truth table is worth 4 points, and there is no partial credit on the truth table. You can't get the SOP form of F right, unless the truth table is right.)

E S	F
00	1
01	1
10	0
11	1

$F = \overline{E} + S$

This problem illustrates a very important concept in logical reasoning. It is called "implication." If-then statements (like, "If E, then S") are implications. The function F is true, if it is consistent with the statement "E *implies* S." It is false, if and only if it contradicts that statement.

Thus, this problem shows that "If E, then S." is logically equivalent to the statement, "Not-E OR S."