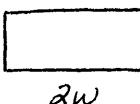
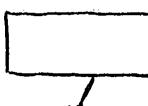
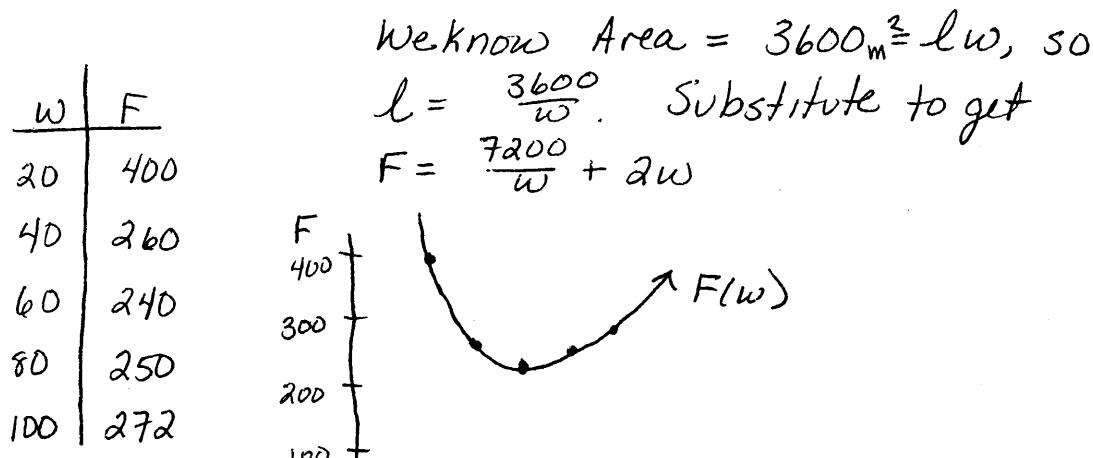


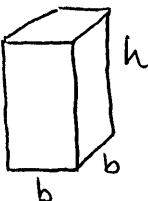
Chapter 1, Section 4

2.  w $2w$ Area = length \times width
 $= w \cdot 2w$
 $A = 2w^2$

6.  w l Suppose length of fence = F .
 $F = 2l + 2w$.



Looks like F is smallest if $w = 60$. Then $l = \frac{3600}{60} = 60$.
The dimensions of approximately $60\text{m} \times 60\text{m}$ should require the smallest amount of fencing.

⑧  h Volume = $1500 \text{ in}^3 = b^2h$. Find surface area in terms of b .

$SA = b^2 + b^2 + bh + bh + bh + bh$

$SA = 2b^2 + 4bh$

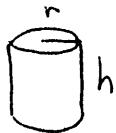
Need to eliminate h . $h = \frac{1500}{b^2}$. Substitute.

$SA = 2b^2 + 4b\left(\frac{1500}{b^2}\right)$

$SA = 2b^2 + \frac{6000}{b}$

Chapter 1, Section 4

(13)

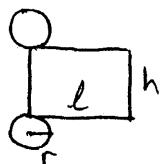


cost for side = 0.02 cents/in².

cost for top & bottom = 0.04 cents/in².

$$\text{Volume} = 4\pi \text{ in}^3 = \pi r^2 h.$$

If we cut open the can, we can lay it flat like this:



$$\text{Cost} = 0.02 (\text{area of rectangle})$$

$$+ 0.04(2)(\text{area of one circle}).$$

We need the length of the rectangle, which is actually the circumference of the circle.

$$l = 2\pi r.$$

$$\text{Cost} = 0.02(2\pi r)(h) + 0.08(\pi r^2).$$

We want to eliminate h so Cost is only in terms of r.

We know Volume = $4\pi = \pi r^2 h$, so $h = 4/r^2$.

$$\text{Cost} = 0.04\pi r (4/r^2) + 0.08\pi r^2$$

$$\text{Cost} = 0.16\pi/r + 0.08\pi r^2$$

15. Population grows at a rate proportional to size of population.
rate of growth $r = kp$ where k is constant and p is the population size.

17. t = temp of object. r = rate temp changes. a = temperature of surrounding medium (this is called the ambient temp).
 $r = k(t - a)$

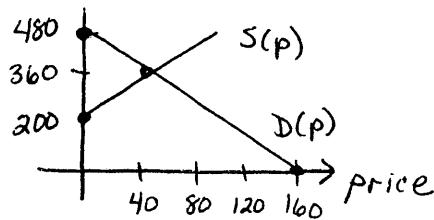
(This particular relationship is called "Newton's Law of Cooling").

Chapter 1, Section 4

- (35.) $S(p) = 4p + 200$, $D(p) = -3p + 480$. Equilibrium is where $S(p) = D(p)$, so solve $4p + 200 = -3p + 480$.
- $$7p = 280$$
- $$p = 40.$$

When $p = 40$, $S(40) = 360$ units and $D(p) = 360$.

(Not a coincidence! The idea of equilibrium means $S=D$).



38. At a speed of 72 kph, after 40 minutes, our hero has a lead of
 $40 \text{ min} \times \frac{1 \text{ hour}}{60 \text{ min}} \times \frac{72 \text{ km}}{1 \text{ hour}} = 48 \text{ kilometers.}$

Total distance is 83.8 km, so he still must travel 35.8 km.

This will take him $35.8 \text{ km} \times \frac{1 \text{ hour}}{72 \text{ km}} = 0.497\bar{2} \text{ hours}$

The pursuers have to travel 83.8 km at 168 kph,
 which will take them $83.8 \text{ km} \times \frac{1 \text{ hour}}{168 \text{ km}} \approx 0.4988 \text{ hours.}$

When our hero reaches the border, the bad guys are just barely behind him, so he gets away.

44. Let $x = \# \text{ checks written per month}$. Cost at bank 1 is $C_1 = 12 + 0.10x$, and at bank 2 is $C_2 = 10 + 0.14x$.

1st bank is better if $12 + 0.10x < 10 + 0.14x$

$$2 < .04x$$

$$50 < x.$$

1st bank is better if you write more than 50 checks per month.

If you write less than 50, 2nd bank is better.

Chapter 1, Section 5

$$1. \lim_{x \rightarrow a} f(x) = b$$

4. $\lim_{x \rightarrow a} f(x)$ does not exist

$$2. \lim_{x \rightarrow a} f(x) \neq b$$

5. $\lim_{x \rightarrow a} f(x)$ does not exist

$$3. \lim_{x \rightarrow a} f(x) \neq b$$

$$6. \lim_{x \rightarrow a} f(x) = b$$

$$10. \lim_{x \rightarrow 0} (1 - 5x^3) = \lim_{x \rightarrow 0} 1 - 5(\lim_{x \rightarrow 0} x)^3 \text{ since it's a polynomial.}$$

$$= 1 - 5(0) = 1.$$

$$(14) \lim_{x \rightarrow 1} \frac{2x+3}{x+1} = \frac{2+3}{1+1} = \frac{5}{2}$$

$$18. \lim_{x \rightarrow 3} \frac{9-x^2}{x-3} = \lim_{x \rightarrow 3} \frac{(3-x)(3+x)}{x-3} = \lim_{x \rightarrow 3} -(3+x) = -6$$

$$(24) \lim_{x \rightarrow 1} \frac{x^2+4x-5}{x^2-1} = \lim_{x \rightarrow 1} \frac{(x+5)(x-1)}{(x+1)(x-1)} = \lim_{x \rightarrow 1} \frac{x+5}{x+1} = \frac{6}{2} = 3$$

$$26. \lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x \rightarrow 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} = \lim_{x \rightarrow 9} \frac{x-9}{(x-9)(\sqrt{x}+3)}$$

$$= \lim_{x \rightarrow 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$$

$$28. \lim_{x \rightarrow 5^-} \frac{\sqrt{2x-1} - 3}{x-5} = \lim_{x \rightarrow 5^-} \frac{(\sqrt{2x-1} - 3) \cdot (\sqrt{2x-1} + 3)}{(x-5) \cdot (\sqrt{2x-1} + 3)}$$

$$= \lim_{x \rightarrow 5^-} \frac{2x-1-9}{(x-5)(\sqrt{2x-1} + 3)} = \lim_{x \rightarrow 5^-} \frac{2(x-5)}{(x-5)(\sqrt{2x-1} + 3)}$$

$$= \lim_{x \rightarrow 5^-} \frac{2}{\sqrt{2x-1} + 3} = \frac{2}{\sqrt{9}+3} = \frac{2}{6} = \frac{1}{3}$$

Chapter 1, Section 5

(30) $f(x) = \begin{cases} \frac{1}{x-1} & x < -1 \\ x^2 + 2x & x \geq -1 \end{cases}$

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} \frac{1}{x-1} = -\frac{1}{2}$$

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} (x^2 + 2x) = 1 - 2 = -1$$

31.

x	1	0.1	0.01	0.001	0.0001
$1000(1 + 0.09x)^{\frac{1}{x}}$	1090	1093.73	1094.13	1094.17	1094.17

37. ~~$\lim_{x \rightarrow 0} f(x)$ does not exist, keeps bouncing between 1 and -1.~~

38. ~~$\lim_{x \rightarrow 0} g(x) = 0$ since graph gets shorter and shorter as $x \rightarrow 0$.~~

Chapter 1, Section 6

4. $f(x) = \frac{2x-4}{3x-2}$. This is continuous at $x=2$: $f(2) = \frac{0}{4} = 0$,
 $\lim_{x \rightarrow 2} f(x) = 0$. (The discontinuity is at $x=\frac{2}{3}$).

(6) $f(x) = \frac{2x+1}{3x-6}$. This is discontinuous at $x=2$ since
 $f(2) = \frac{5}{0}$ and is not defined.

10. $f(x) = \begin{cases} x+1 & x < 0 \\ x-1 & x \geq 0 \end{cases}$ $f(0) = 0-1 = -1$.
 $\lim_{x \rightarrow 0^+} f(x) = 0-1 = -1$ } not continuous
 $\lim_{x \rightarrow 0^-} f(x) = 0+1 = 1$ } at $x=0$,

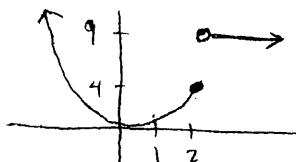
(11) $f(x) = \begin{cases} x^2+1 & x \leq 3 \\ 2x+4 & x > 3 \end{cases}$ $f(3) = 3^2+1 = 10$
 $\lim_{x \rightarrow 3^+} f(x) = 2(3)+4 = 10$ } continuous
 $\lim_{x \rightarrow 3^-} f(x) = 3^2+1 = 10$ } at $x=3$

14. $f(x) = x^5 - x^3$ is continuous everywhere.

18. $f(x) = \frac{x^2-1}{x+1}$ is discontinuous at $x=-1$

22. $f(x) = \frac{x^2-2x+1}{x^2-x-2} = \frac{(x-1)(x-1)}{(x-2)(x+1)}$ discontinuous at $x=2, -1$.

(24) $f(x) = \begin{cases} x^2 & x \leq 2 \\ 9 & x > 2 \end{cases}$ Discontinuous at $x=2$: $\lim_{x \rightarrow 2^-} f(x) = 2^2 = 4$
 $\lim_{x \rightarrow 2^+} f(x) = 9$ ↗
Graph "breaks" at $x=2$.



Chapter 1, Section 6

32. $f(x) = x(1 + \frac{1}{x})$ is continuous on $0 < x < 1$, but $f(x)$ does not exist at $x=0$, so on $0 \leq x \leq 1$, $f(x)$ is discontinuous at $x=0$.

35. Show that $\sqrt[3]{x} = x^2 + 2x + 1$ must have a solution on $0 \leq x \leq 1$.

Define $f(x) = x^2 + 2x - \sqrt[3]{x}$. For original equation to have a solution, we need $f(x) = 0$.

$$f(0) = -1 \text{ and } f(1) = 1$$

since $-1 < 0 < 1$, and

$\frac{-1}{1}$ must cross somewhere
in between!

$f(0) = -1$, $f(1) = 1$, using the intermediate value property there is some number c so that $0 < c < 1$ and $f(c) = 0$. (Since $f(x)$ is continuous).

38. The minute hand of a clock moves in a continuous fashion. Since the hour hand moves much slower, there is a time when the minute hand is behind the hour hand, as well as a time when it's ahead. Since the motion is continuous, there must also be a time when the hands coincide.