Technical and Computational Aspects of the Measurement of Aggregate Shape by Digital Image Analysis

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Abstract: Aggregates need to pass numerous tests to ensure the performance of asphalt and concrete structures and pavements. Some of these tests are fairly onerous, requiring manual, labor intensive, cost ineffective measurements that do not provide significant statistical validity, and are prone to errors through ignorance, negligence, or even in some cases through deliberate misrepresentation. This paper presents a vision based alternative to measure the shape of aggregate particles. The system, although requiring increased capital investment, will result in objective, cost effective, and timely testing of aggregate shape. The system uses dual, synchronized, double speed progressive scan cameras to image the aggregate piece from two directions. A dual image acquisition card simultaneously digitizes both images and does real-time thresholding to create a binary image, which is ported to the host computer. A software trigger determines the presence of an aggregate piece in the image, and the boundaries of the piece are delineated by a perimeter-walking routine. Measurements of aspect ratio and minimum curve radius are made on the perimeter array, and are compared to flat and elongated tests, coarse aggregate angularity (uncompacted voids), compacted voids, and fractured face counts.

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Introduction

Aggregates encompass the bulk of the materials used in highway construction; consequently the performance of the concrete or asphalt pavement relies heavily on the performance of the aggregate materials. Careful selection of these materials, with respect to stringent standards, ensures that the pavements will perform as designed, and not suffer from premature deterioration.

Aggregates consequently must pass a stringent series of mechanical, chemical and physical tests in order to demonstrate that they will perform satisfactorily, and meet or exceed specifications. Several of the required physical tests characterize the shape of aggregates. The importance of aggregate shape measurements like flat and elongation, fine aggregate angularity and coarse aggregate angularity in concrete and asphalt pavements have recently been realized, most notably in flexible pavement. This new understanding is reflected in the new Superpave (Superior Performing Asphalt Pavements) guidelines developed as a product of the Strategic Highway Research Program.

Test procedures for many aggregate tests have been well established and are specified, for example, by American Society of Testing and Materials (ASTM) standards, American Association of State Highway and Transportation Officials (AASHTO) standards, or Superpave guidelines. In most cases the methods of testing are well accepted by the industry, however physical testing has always been a time-consuming, tedious and labor-intensive process. Consequently these types of tests are often performed reluctantly and infrequently, resulting in test results that are at best done infrequently and perhaps not statistically representative.

The alternative is to use image analyzing computers that can efficiently make objective test measurements.

Flatness and Elongation

The measurement of flat and elongated is a measure of the aspect ratios of a particle. D'Angelo (1996) summarizes the problems with flat and elongated particles. The new Superpave guidelines call for no more than 10% of coarse aggregates to have aspect ratios greater than 5:1, and there is a possibility of a new standard at 3:1.

Currently manual measurements using calipers are employed for flat and elongated (ASTM 1995) (Fig. 1). These measurements are not only slow and laborious, but are also highly subjective. The method involves first obtaining a sample and screening it into the various size fractions. Each piece then needs to be manually handled, by first passing it through one side of the proportional caliper, then rotating it and passing it through the other side. Based on whether the piece passes through the caliper or not it is placed in a separate pile. Each pile, for each size fraction needs to be weighed, and tabulated. If more than one aspect ratio is required, then this procedure must be repeated for each aspect ratio to be measured (3:1, 5:1). If flatness and/or elongation are to be measured individually, the entire task has to be repeated. This task is tedious, and consequently there is a potential for poor implementation and inaccurate results. Because it is labor intensive and time consuming, it is costly. A typical sample may take up to 1 h for analysis, and results may not be available for several hours, during which hundreds of tons of material may be produced that do not meet specifications. Because they are so inefficient and expensive, often there are not enough measurements to produce a statistically valid sample.

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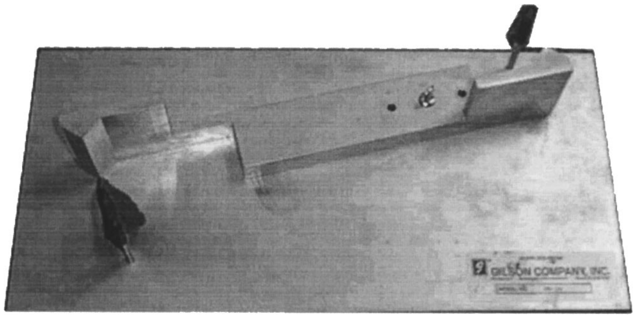


Fig. 1. Proportional caliper for measuring flat and elongated particles

Coarse Aggregate Angularity, and Fractured Face Counts

Three physical aggregate tests deal with aggregate angularity. Angularity is the measure of the ''roundness'' of an aggregate piece D'Angelo (1996) summarizes the issues with angularity. The tests that are used to characterize aggregate angularity are compacted voids test (ASTM 2000a) and uncompacted voids tests (ASSHTO 2002; ASTM 2000b) (which assume a correlation between angularity and void ratio), and fractured face counts (ASTM 2001). The voids tests are laboratory intensive tests, which require dried and screened aggregate to be dropped (Fig. 2) into a container or compacted into a container. The void ratio is then calculated using the container volume, total aggregate weight, and aggregate density.

The fractured face count test is a simple manual count of the percentage of coarse aggregate with one or two or more fractured faces. Superpave guidelines call for up to 100% of the material with two or more fractured faces, depending on traffic density.

Both styles of tests are tedious and time consuming. The voids test assume a relationship between void ratio and angularity, while the fractured face count is very objective, because it is often difficult to define a fractured face.

Impact of Image-Based Measurements

The impacts of a successful image based methodology are numerous:

1. Test results, removed from human subjectivity, will be much more reliable. No longer will the test results vary between operators, or vary based on the disposition of an operator.
2. A greater number of tests will be performed. Faster testing, and the low per unit cost of incremental tests, will result in an increased amount of tests being conducted, allowing better and more statistically valid characterization.
3. Run time adjustments to crushing, screening and other processing equipment will be possible. Because the analysis is quick, a significant reduction of off-specification material can be achieved, and there will be less incentive to pass off-specification material.
4. There will be a lower burden on operators and testing agencies, resulting from lower per sample testing costs.

However there are also difficulties with image based measurement methodologies:

1. The capital costs of imaging equipment will be much higher;
2. Inherent small to significant differences in measurement results can be expected, because of the differences between imaging and physical testing techniques; and
3. Industry and regulatory resistance can be expected to any new technology that does not give exactly the same results as the ''older'' manual measurements, even if the ''older'' measurements are less accurate.

State of Art in Image-Based Measurements

The imaging system-described in this paper, and its previous development have been introduced in Maerz (1998); Maerz and Zhou (1999); and Maerz and Lusher (2000).

Other authors have proposed and or built similar systems. Barksdale et al. (1991) researched the possibility of using modern data acquisition procedures to measure aggregate. Kuo et al. (1996; 1998) and Frost and Lai (1996) developed a method to analyze the morphological characteristics of coarse aggregate using a three-dimensional (3D) image analysis process with aggregates in Plexiglas holders. Brzezicki and Kasperkiewicz (1999) improved on this concept by measuring the shadows along with the aggregate particle at perpendicular projections, enabling three-dimensional characteristics to be measured. Prowell and Weingart (1999) evaluated the precision of the VDG-40 (a granulometry instrument) in measuring flat and elongated particles. Rao and Tutumluer (2000) developed an image analyzing system using three cameras at orthogonal views to measure the volume of an aggregate as well as the aspect ratios. Similar research was conducted by Masad et al. (2001). A laser based scanning system has been proposed by Kim et al. (2000).

Shape Measurement

There are three independent measures that can be considered as indicators of particle shape. These are aspect ratio (form, spheric-

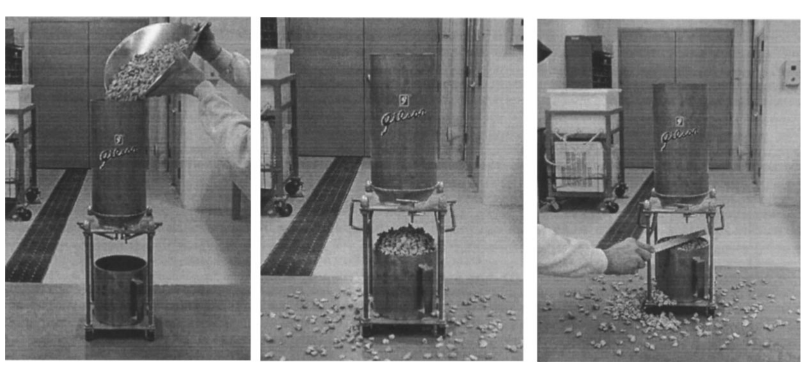


Fig. 2. Uncompacted voids test for measuring void ratio/coarse aggregate angularity

ity), roundness (angularity), and surface texture (particle texture) (Masad et al. 2001). Historically only aspect ratio and roundness have been considered to be indicators of shape (Pettijohn 1949; Krumbein and Sloss 1951). Similarly modern texts use the same distinction (National Sand, Stone, and Gravel Association 1991; Smith and Collis 2001).

In addition, texture is much more difficult to measure, and requires measurement at a different scale of observation. Consequently this discussion of aggregate shape will be limited to aspect ratio and roundness.

Aspect Ratio

Ostensibly aspect ratio is easy to visualize and easy to measure in a regular-shaped particle. The ''box principle'' uses the idea of fitting a 3D box around the aggregate piece and recording the size of the box in terms of length, width and height. Thus the dimensions are the maximum in three mutually perpendicular directions, with the long axis of the particle aligned with the long axis of the box.

The proportional caliper measurement of flat and elongated deviates from this because the measurements are taken independent of orientation. The minimum dimension is not necessarily perpendicular to the maximum or intermediate diameter. Also in screening, nominal particle size (size of square opening that a particle will pass) depends on intermediate diameter, but is not a true measure of intermediate diameter, as flattened pieces can pass through the diagonal of the square screen.

For the imaging analysis the box principle is used. The principals of mutual orthogonality and box size are preserved.

Roundness

There are a number of shape parameters pertaining to roundness in the literature. A review is given by Franklin (1996a,b). Pettijohn (1949) and Krumbein and Sloss (1951) make use of inscribed circles fitted into the corners (vertices) of the projection of the particles. This is the technique used for the imaging analysis.

Imaging System: Hardware Description

Sample Presentation

Imaging systems require the individual aggregate pieces be transported to where they can be imaged by the camera, and then moved out of the way so that others can be imaged. Furthermore, for the purpose of 3D imaging, pieces must be imaged individually so that the same piece can be unambiguously identified in both views. Two prototype devices have been designed a black miniconveyor, and a translucent rotating table.

Black Miniconveyor Belt

The first prototype sample presentation (Fig. 3) uses a black belt to create a contrast between the sample and background. It features a vibrating feeder, black transport belt, and chute discharge. A black backdrop is added and small lamps used for variable intensity and variable angle illumination. Cameras are mounted on extension arms.

The belt has an operational speed of about 0.055 m/s and a maximum speed of 0.18 m/s. The vibrating feeder has an adjustable cam to increase or decrease vibration. The vibration level is typically set in conjunction with the belt speed so that there is at least a 2 in. separation of particles when they are placed on the belt.

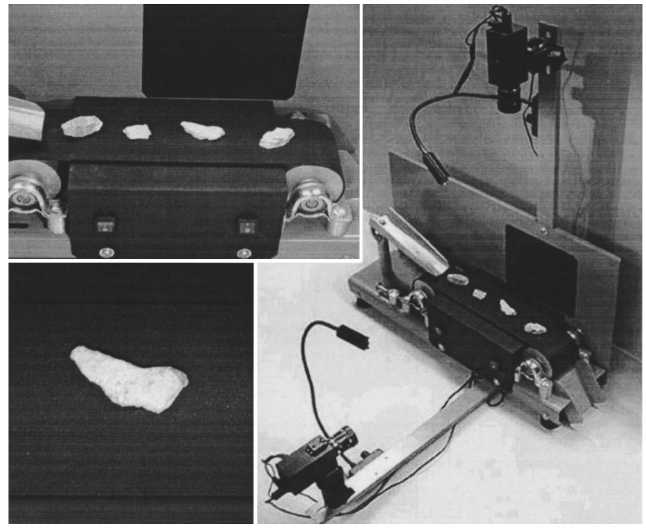


Fig. 3. First prototype of WipShape shape measurement system, using mini-conveyor belt

This device works well for, and is still being used for light colored aggregates, but it is difficult to maintain the contrast with darker or mottled aggregates. This is because even a black belt reflects significant amounts of light.

Translucent Rotating Table

The final prototype (Fig. 4) uses a translucent rotating table to create contrast by backlighting and imaging a silhouette. It features a vibrating feeder, circular translucent transport table, and a rotating brush for discharge. Fiber optic backlighting is used for variable illumination. Cameras are mounted on extension arms.

The table has an operational speed (at the center of the image) of about 0.080 m/s (10.9 rpm) and a maximum speed of 0.15 m/s (20.0 rpm). The vibrating feeder is the same as for the black miniconveyor.

The device works well for all aggregates, however very light colored aggregates are sometimes difficult to silhouette completely. This is because there is cross contamination of light. The

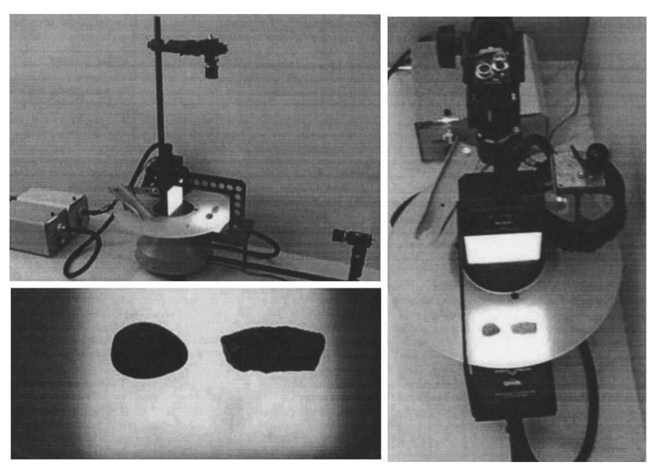


Fig. 4. Final prototype of WipShape shape measurement system, using translucent rotating table

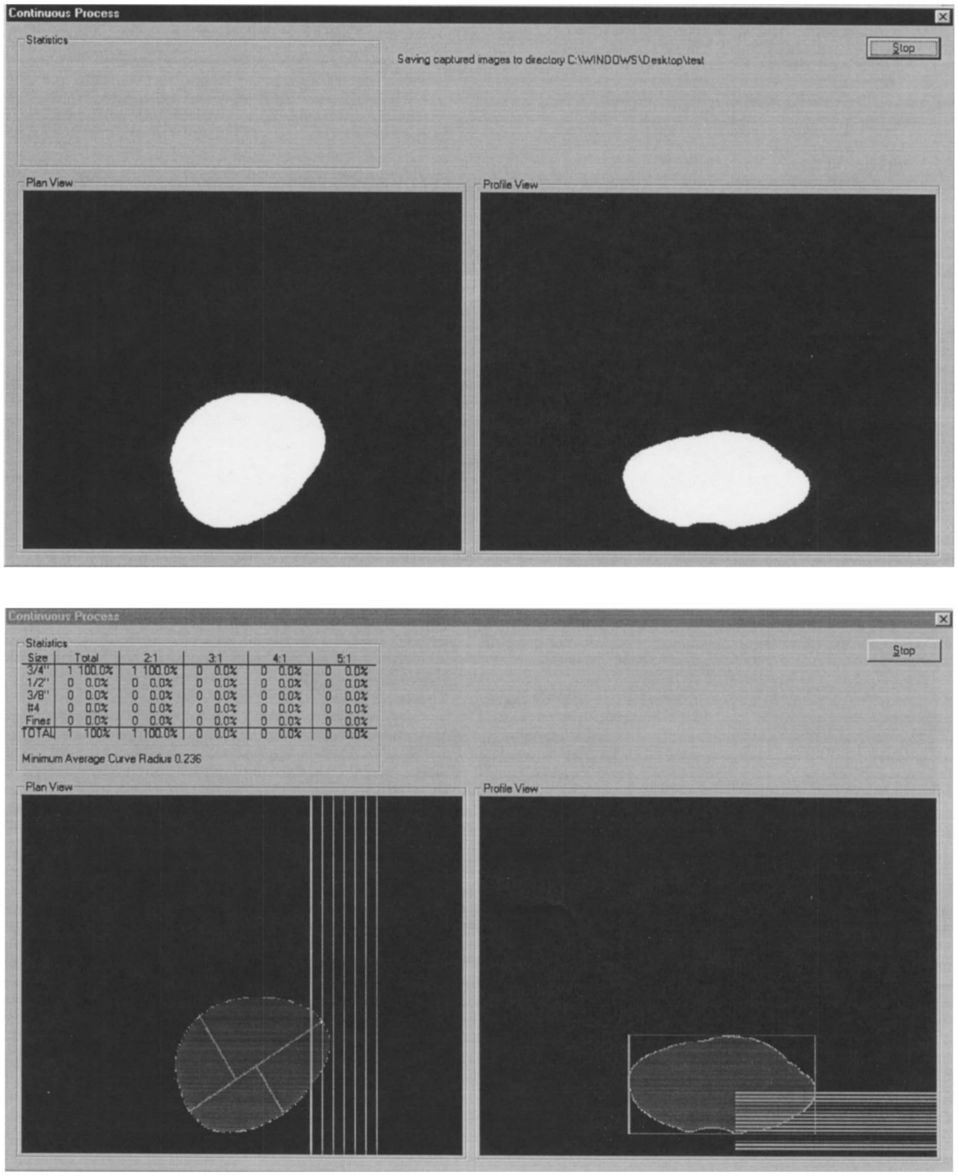


Fig. 5. Analysis of rounded particle: (top) raw captured binary image, plan, and profile view; (bottom) analyzed image, parallel lines are trigger searches, lines in and around pieces define dimensions of particle

light that silhouettes the fragment in the profile view also reflects light off the fragment in plan view.

Imaging

Cameras

The cameras used for this application are a pair of Sentech STC-1000 progressive scan (noninterlaced), double speed cameras. The cameras are synchronized with each other, so they are imaging simultaneously at 60 frames/s.

Framegrabber

The digitization board is an Imaging Source DFG-BW1. This board simultaneously digitizes images from both cameras, and provides the power and triggers to the cameras. On board look up tables allow real time thresholding. This produces the binary image required (2 bits per pixel), and reduces the bandwidth required for transferring images.

Imaging System: Software Description

The software application is developed as a *Windows*® application under *Using Visual c++*®.

Image Acquisition Loop

Image acquisition is facilitated by a capture software development kit provided by manufacturer of the digitization board. Fig. 5 shows a dual binary image as it is captured and brought to the image buffer in the application. The acquisition board provides for lookup tables which allow real time thresholding; a binary (black and white, 1 bit per pixel) image is transferred from the acquisition board to the host buffer.

Image acquisition is a continuous loop, in real time. A software trigger (triggered by the presence of a block under the vertical and horizontal parallel lines in the image in Fig. 5) looking for five or more contiguous white pixels along the trigger

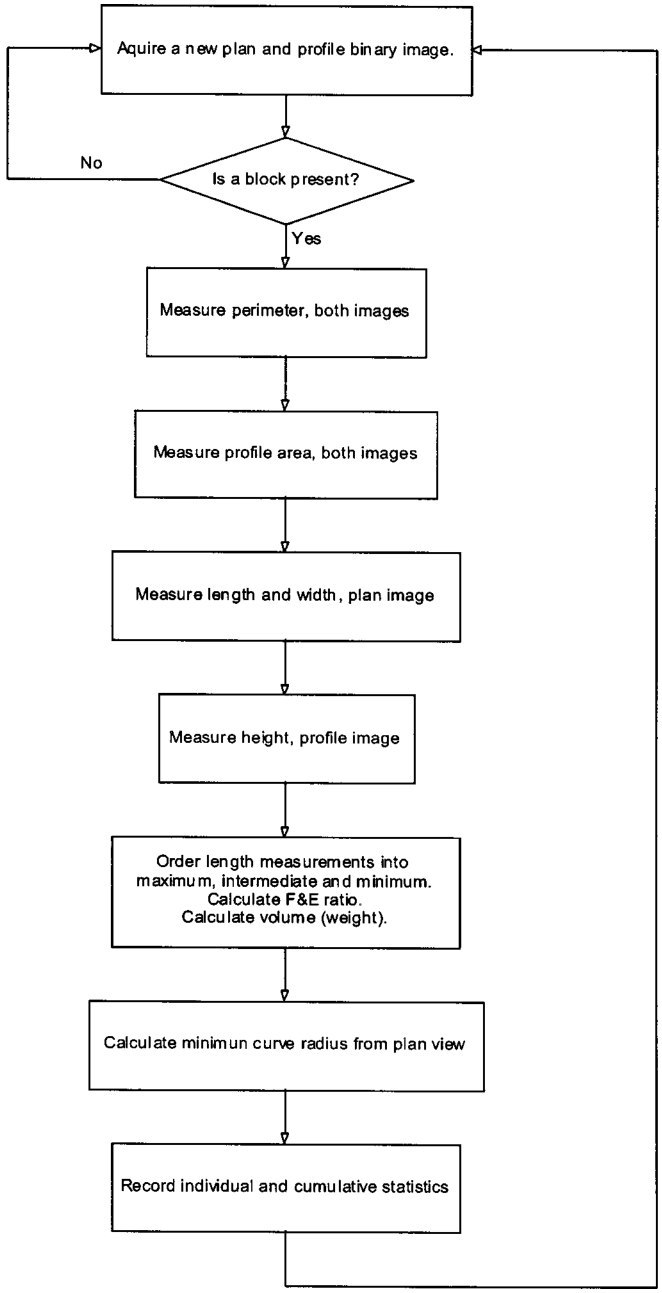


Fig. 6. Flow chart of measurement operations

lines, determines that a block is present in both views. If a block is found it is analyzed as below. If no block is found, the next set of images is acquired.

Measurements

Working on the binary image (Fig. 5), the following operations are done (Fig. 6 shows a flow chart):

1. A perimeter walk creates an array of $x-y$ coordinates defining the outline of each view of the block. The algorithm consists of moving a 3×3 pixel mask along the perimeter of the particle (following the black/white interface on the image). The perimeter is painted in yellow, and placed into a vector array of perimeter coordinates.
2. A recursive pixel filling (paint) routine calculates the profile surface area of each view of the block. The area is painted in red.

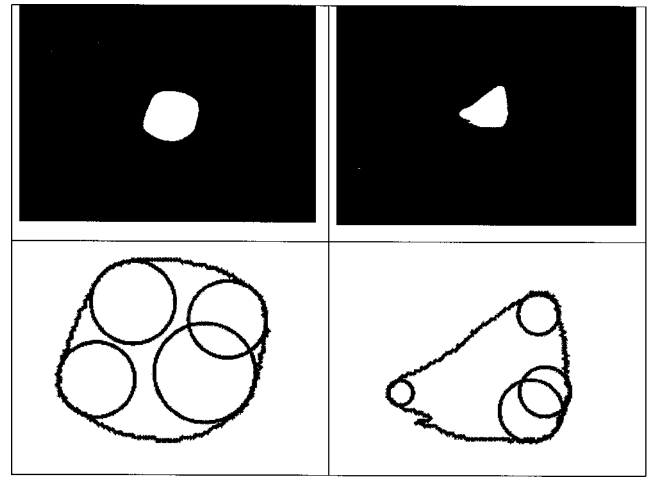


Fig. 7. Average minimum curve radius calculations: (left) rounded aggregate; (right) angular aggregate; (bottom) aggregate profiles with inscribed curve radii, as measured by algorithm

3. In the plan view, using the perimeter array, the longest dimension (major axis, length) is identified and measured as the length of the aggregate. The algorithm calculates the distance between each point in the perimeter array. The line between two points that are the farthest apart defined as the maximum dimension (length). That line is recorded as the length and painted in green.
4. Also in the plan view, the longest half width on each side of and perpendicular to the major axis is identified and measured. The algorithm measure the perpendicular distance between each point on one side of the particle and the line defining the length, for both sides. Adding both lengths together gives the width of the aggregate. (This strategy was chosen as these two measurements, the length and width, define a rectangular box with the length parallel to the sides of the box.) Both lines are painted in green.
5. In the profile view, using the perimeter array, the maximum height of the particle is identified and measured by measuring the difference in length between the highest point and lowest parts of the profile. The box encompassing the block is painted in green.
6. If the measured length is not greater than the width, and the width is not greater than the height, the measured lengths are re-ordered. (This might be the case if a particle was laying on its side or end, rather than on the flattest surface.)

Sizing

The size of the aggregate is taken to be the width of the particle. This is to provide compatibility with screening results (It is primarily the width which governs the minimum screen size that a particle can pass through.) An empirical calibration factor is used to match screening size measurements. It was experimentally determined that best estimate of particle volume could be determined by the following equation:

$$[\text{volume} = \text{length} \times \text{width} \times \text{height} \times 0.8] \quad (1)$$

This reflects the fact that statistically, the typical particle observed occupies about 80% of the volume of a rectangular parallelepiped of the same length, width, and height. While this relationship will vary depending on the degree of rounding of the edges, it is

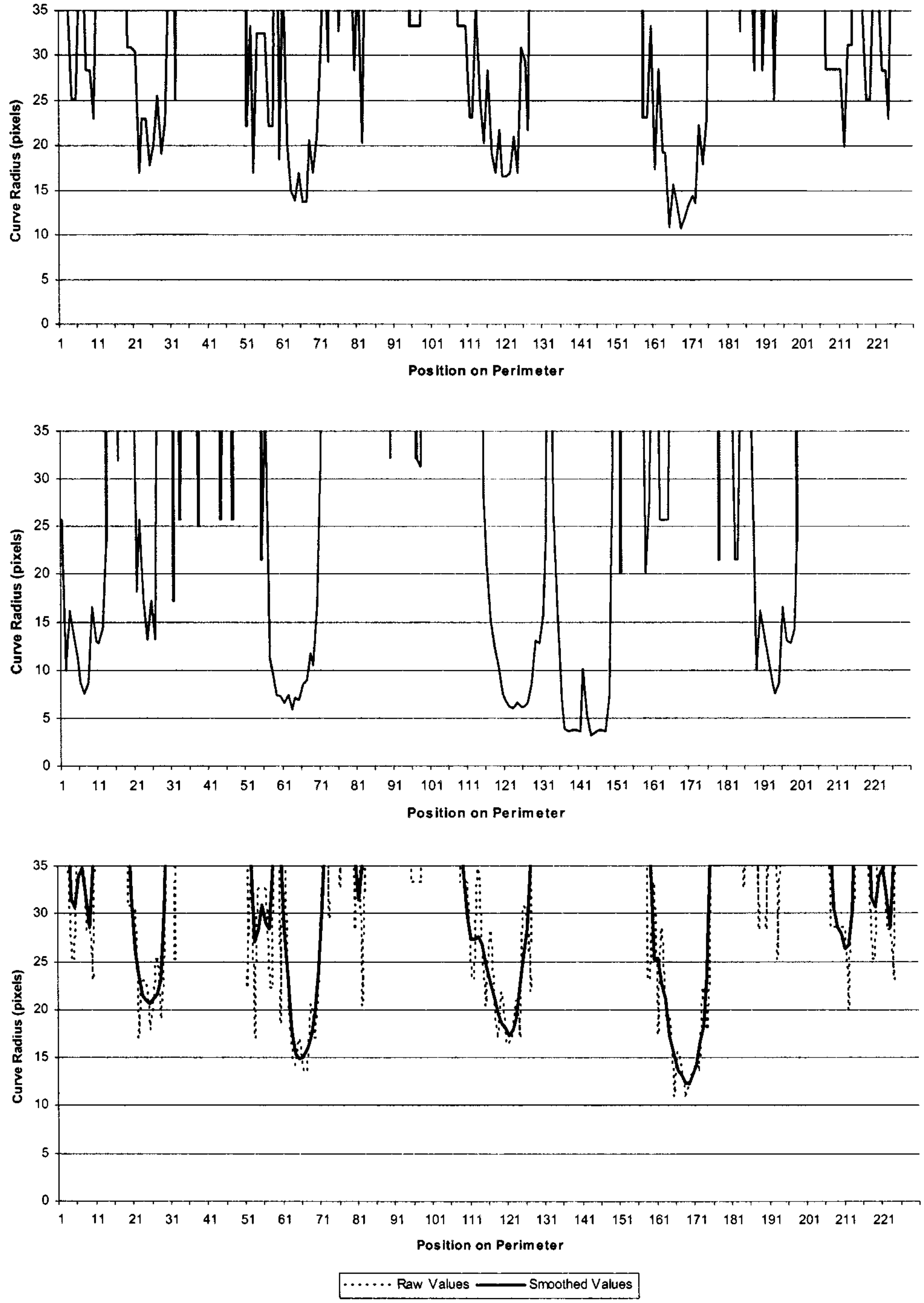


Fig. 8. Curve radius measurements around profiles of Fig. 7: (top) rounded aggregate; (middle) angular aggregate; (bottom) curve radius of rounded particle; raw and smoothed values

irrelevant in the final analysis as results are normalized by weight percent.

Aspect Ratio

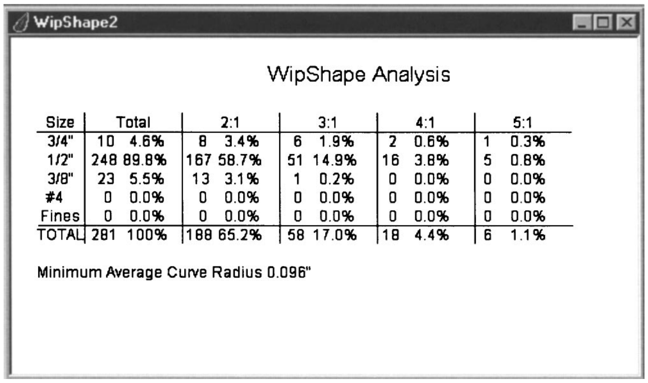
The aspect ratio is determined by dividing the maximum dimension (length) by the minimum dimension (height). Particles are classified as being greater than 5:1, 4:1, 3:1, 2:1 or 1:1.

Angularity

The aggregate angularity is defined as the minimum average curve radius of the individual particles is measured as illustrated in Fig. 7.

Angularity is calculated in the following way:

1. From the perimeter array, the radius of a circle containing three points on the profile is calculated, each point separated by ten pixels along the perimeter (a ten pixel separation was determined to be small enough to be sensitive to small curves, but large enough to be unaffected by the noise and aliasing along the perimeter). An instantaneous curve radius is determined for each point on the profile in this manner, creating an array of curve radii (Fig. 8).
2. The array of curve radii values are smoothed by a moving average filter. A five-point Gaussian low pass filter is used (Fig. 8).


WipShape2

WipShape Analysis

Size	Total		2:1		3:1		4:1		5:1	
3/4"	10	4.6%	8	3.4%	6	1.9%	2	0.6%	1	0.3%
1/2"	248	89.8%	167	58.7%	51	14.9%	16	3.8%	5	0.8%
3/8"	23	5.5%	13	3.1%	1	0.2%	0	0.0%	0	0.0%
#4	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%
Fines	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%
TOTAL	281	100%	188	65.2%	58	17.0%	18	4.4%	6	1.1%

Minimum Average Curve Radius 0.096"

Fig. 9. Example output

3. The array of smoothed curve radii is examined to find local minima in the curve radius function.
4. A test is performed to ensure that a corner of the aggregate piece does not result in more than one local minima.
5. The list of local minimum curve radii is ordered from smallest to largest.
6. The four smallest curve radii are averaged to produce the minimum average curve radius of the individual piece. (Four radii are used because one would expect that depending on the shape of an aggregate particle, its profile might typically have anywhere between three to five corners, four being the average.)

Data Output

Data are output as chart of aspect ratio versus size retained (Fig. 9). In each category the numbers of particles found are listed as

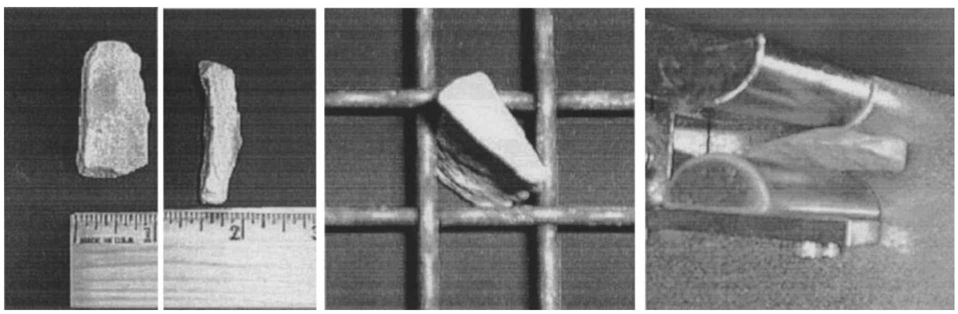


Fig. 10. Example of aggregate piece (left) which has intermediate diameter of 1 in. (as measured by imaging system) but will pass though 3/4 in. screen diagonally because it is so thin (center); Aspect ratio as measured by imaging is 4:1, but it will pass through the proportional caliper at 5:1 setting (right), because it is curved and can be rotated through opening

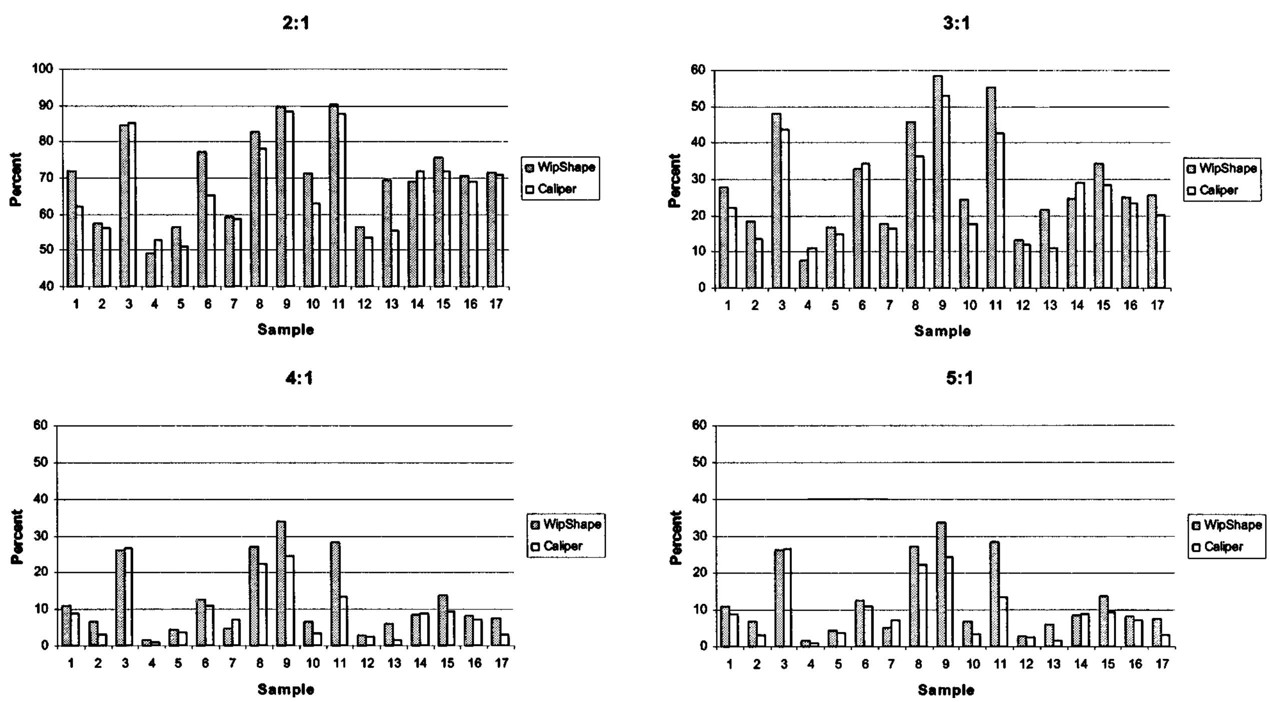


Fig. 11. Comparison of imaging (WipShape) and proportional caliper testing, for aspect ratios of 2:1, 3:1, 4:1, and 5:1

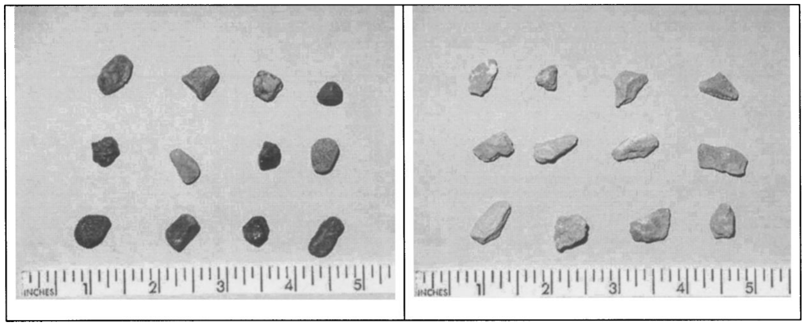


Fig. 12. Example of aggregates, retained on No. 4 screen, passing 3/8 in. screen: (left) rounded river gravel; (right) angular crushed dolomite

well as the weight percent. On the bottom the minimum average curve radius in inches is given.

Performance

A sample of 100 pieces of aggregate takes less than 2 min to process using the imaging system. This compares to 25–30 min for flat and elongated (four different aspect ratios) and more than 15 min for a typical voids test, not counting calculations and report preparation.

Example Results

Measurement Deviations

Measurements made by imaging systems do not always match traditional testing results. Examples are shown in Fig. 10, where measurement differences are simply the result of measuring different things.

In this example a piece which measures 1 3/4 in. by 1 in., by 3/8 in. (when manually measured) is classified as a 3/4 in. (retained) size as a result of imaging. But it is classified as a 1/2 in. retained by screening methods.

Also because of the curved nature of the piece, it can be rotated through the proportional caliper opening set to 5:1, whereas the imaging measurement set the aspect ratio at 4:1.

Flat and Elongated

Fig. 11 show test results comparing imaging measurements and proportional caliper tests. For the caliper tests, 100 pieces of each sample were tested, for the imaging test, 1,000–3,000 pieces were tested. These show mixed results. Most of the imaging measurements are very close to caliper measurements while however a few show a significant deviation. While the exact reason for this is not known, it may be related to the fact that for these few samples the differences in the measurement methods is significant.

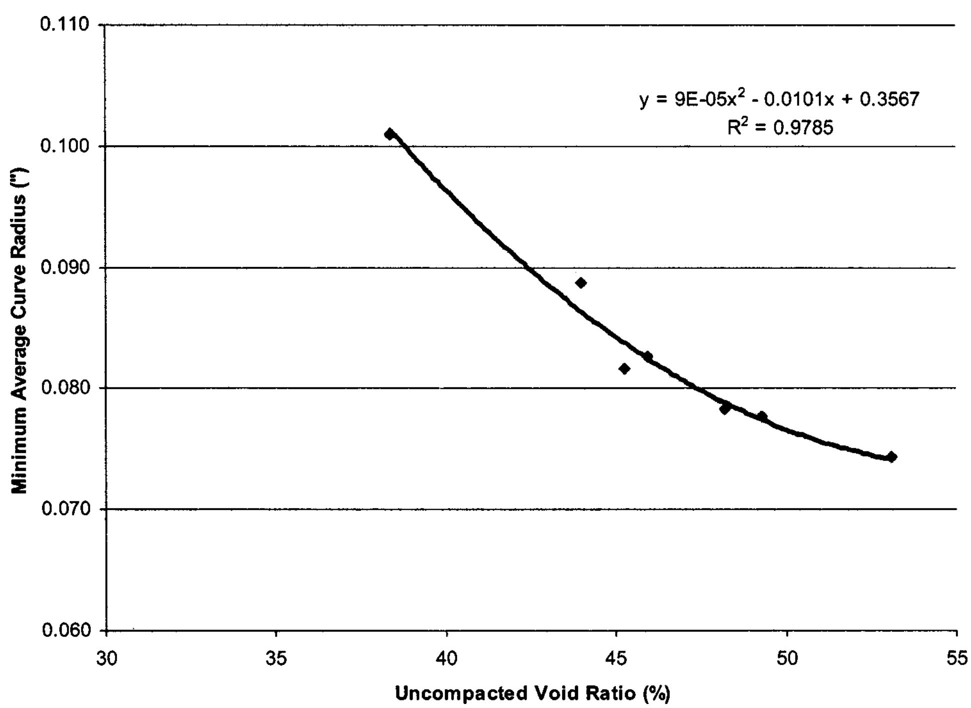


Fig. 13. Graph of uncompacted void ratio versus minimum average curve radius, for seven aggregate samples, retained on No. 4 screen, passing 3/8 in. screen

Coarse Aggregate Angularity

The relationship between the various voids tests, and particle shape is empirical by nature, and so the relationship between minimum average curve radius and void ratio will be empirical as well.

Fig. 12 shows test results using control specimens. A rounded river gravel and crushed dolomite were used, and mixed 50% of each by weight to make a control sample. A good preliminary correlation was found between uncompacted void ratio and minimum average curve radius (Fig. 13). Similar results were found with compacted void ratio and crush counts. This implies that it may be possible to measure coarse aggregate angularity directly, avoid the inefficient mechanical tests, and yet come out with the same results.

Summary

Imaging tests can be used to replace physical tests for shape measurement. The advantages include lower unit costs, less subjectivity, faster results, and the ability to produce a greater number of measurements to increase statistical validity.

Tests have shown that imaging results do not always match those of physical testing. Even though imaging tests may be more accurate and reproducible, it will take some effort to get the industry to accept the new testing procedures.

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