# AnisDep: A FORTRAN program for the estimation of the depth of anisotropy using spatial coherency of shear-wave splitting parameters

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## ABSTRACT

We present a FORTRAN-77 program to estimate the optimal depth of the source of anisotropy using spatial coherency of teleseismic shear-wave splitting (SWS) parameters. For a given assumed depth of anisotropy, the program computes a variation factor which is a weighted sum of the arithmetic standard deviation (SD) of the splitting times and the circular SD of the fast directions over overlapping blocks. The optimal depth of anisotropy corresponds to the minimum variation factor. The program executes computations over different block sizes for testing the stability of the resulting optimal depth. A synthetic shear-wave splitting data set is provided for testing the program. In addition to anisotropy depth estimation, some of the subroutines in AnisDep can be used for other applications such as computing circular mean and SD of the observed fast directions, and for computing the coordinates of ray-piercing points. The program is designed for Linux and Sun Solaris platforms, but can be easily adapted for other platforms.

## 1. Introduction

Over the past 30 years, the polarization direction of the fast shear-wave, $\phi$, and the arrival time difference between the fast and slow waves, $\delta t$, of *teleseismic P-to-S* converted waves at the core-mantle boundary (*XKS*, including *SKS*, *SKKS*, and *PKS*) have been widely used to infer the existence, direction, and strength of mantle fabrics (e.g., Silver, 1996; Savage, 1999; Long and Silver, 2009). Unfortunately, mostly due to the steep incidence of the *XKS* waves, the depth of the anisotropic region responsible for the observed shear-wave splitting (SWS) is poorly constrained, leading to heated debates about the lithospheric or asthenospheric origin of the observed anisotropy.

In a recent study, Liu and Gao (2011) proposed and tested using synthetic and real data a procedure to estimate the optimal depth of the source of anisotropy by measuring the spatial coherency of *XKS* splitting measurements. This *technical* note introduces a FORTRAN-77 program which can be accessed from the on-line supplementary site of the journal.

## 2. The depth-estimation procedure

The procedure was described and tested in Liu and Gao (2011) and is briefly summarized below. A study area is first divided into $dx$ by $dx$ square degree blocks. A variation factor, $F_v$, is computed

at each assumed depth of anisotropy. $F_v$ is the weighted sum of the standard deviations of the observed splitting parameters over all the blocks. A range of $dx$ values are used in order to explore the stability of the resulting optimal depth of anisotropy which corresponds to the minimum $F_v$.

For a given $dx$ value, the procedure includes the following steps:

(1) Dividing the top $dep1$ to $dep2$ km of the Earth into thin layers of $ddep$ km thick.
(2) For each layer, computing the geographic coordinates of the ray-piercing points at the top of the layer.
(3) Dividing the area covered by the ray-piercing points into $dx$ by $dx$ degree$^2$ blocks, and for each block, computing the arithmetic SD of the splitting times and the circular SD of the fast directions with ray-piercing points inside the block, i.e., for the $i$th block

$$F_{\delta t}^{(i)} = \sqrt{\frac{1}{M_i - 1} \sum_{j=1}^{M_i} (\delta t_{ij} - \overline{\delta t_i})^2} \tag{1}$$

$$F_{\phi}^{(i)} = \sqrt{-\frac{1}{2} \ln(R_i)} \tag{2}$$

where

$$R_i^2 = \left[ \frac{1}{M_i} \sum_{j=1}^{M_i} \cos(2\phi_{ij}) \right]^2 + \left[ \frac{1}{M_i} \sum_{j=1}^{M_i} \sin(2\phi_{ij}) \right]^2 \tag{3}$$

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$M_i$ is the number of measurements for the $i$th block, $\phi_{ij}$ and $\delta t_{ij}$ are the $j$th fast direction and splitting delay time measurement in the $i$th block, respectively, and $\overline{\delta t_i}$ is the arithmetic mean over all the splitting time measurements in block $i$. $M_i$ should be $\geq 2$. Eq. (2) is based on Eqs. (2.3.5) and (2.3.14) of Mardia (1972) with $l=2$, and Eq. (3) is based on Eqs. (2.2.3) and (2.2.4). While it is technically feasible to weigh the individual measurements by the reciprocal of the standard deviations of the individual SWS measurements in the equations above, we choose not to do so because of the well-known strong dependence of the standard deviations of the individual measurements on the beginning and end of the *XKS* window, the filtering parameters, and the SWS measuring techniques (Vecsey et al., 2008). Such dependence leads to unreliably determined standard deviations of the individual events, which might introduce errors in the resulting variation factors.

(4) Computing the variation factor, $F_v^{(i)}$, for the the $i$th block as a dimensionless weighted sum of $F_\phi^{(i)}$ and $F_{\delta t}^{(i)}$, i.e.,

$$F_v^{(i)} = w_\phi F_\phi^{(i)} + w_{\delta t} F_{\delta t}^{(i)} \tag{4}$$

where $w_\phi$ and $w_{\delta t}$ are the weighting factor for the $\phi$ and $\delta t$ measurements, respectively. Because the $\phi$ measurements have a maximum range of variation of $180°$ and the $\delta t$ measurements have a range of about 2.0 s for most studies (Long and Silver, 2009), we use $w_\phi = 1/180\,\text{degree}^{-1}$ and $w_{\delta t} = 1/2\,\text{s}^{-1}$ so that the variation factors from the two sets of measurements can be combined.

(5) Computing $F_v$ and its SD at a given depth as the arithmetic mean and SD of $F_v^{(i)}$ over all the blocks, i.e,

$$F_v = \frac{1}{N}\sum_{i=1}^{N} F_v^{(i)} \tag{5}$$

$$\sigma_{F_v} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(F_v^{(i)} - F_v)^2} \tag{6}$$

where $N$ is the number of blocks.

The optimal depth corresponds to the minimum $F_v$ value which can be observed on the diagram produced by a GMT (Generic Mapping Tools, Wessel and Smith, 1991) program provided in the package (plot.gmt, see below), or any other plotting programs.

## 3. Usage of AnisDep

The original version of AnisDep contained many subroutines residing in various folders in the authors' local computer. For easy handling and use, we have merged all the subroutines into a single file consisting of a main program and about 40 subroutines/external functions. The FORTRAN-77 program has about 2900 lines (including comments and empty lines).

The program (AnisDep.f), the Makefile, the test data set (test.dat), and all the other necessary files are bundled in a Linux tar file named AnisDep.tar. After the tar file is transferred to the user's local Linux or Unix computer from the journal's electronic supplement site, the files can be extracted using *tar -xf AnisDep.-tar*. The program can then be compiled by typing *make*, and be executed by typing *AnisDep.exe* and entering the name of the input file containing the measured splitting parameters. A synthetic data set, test.dat, can be used to test the program and to serve as an example for the input file. The program displays the *dx* value, the latitudes/longitudes of the area covered by the ray-piercing points at a given depth, and the depth, $F_v$, $\sigma_{F_v}$,
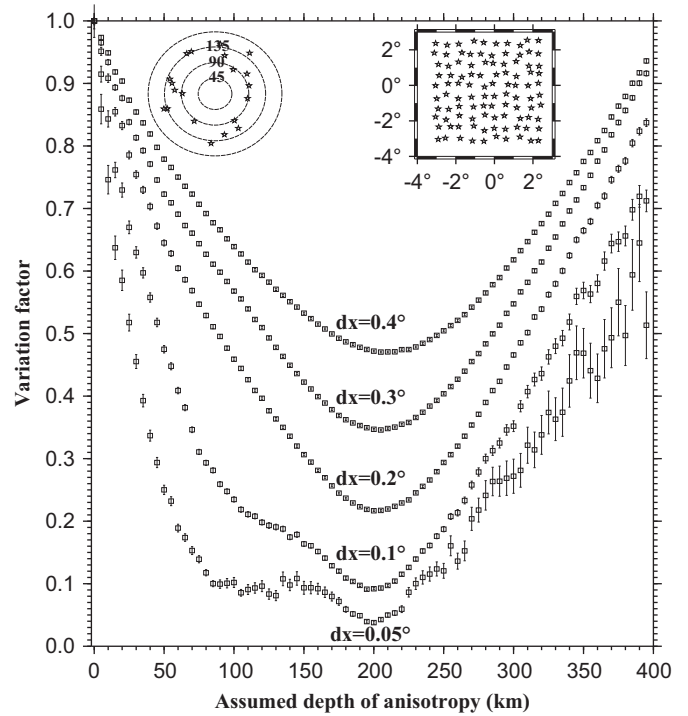


Fig. 1. Spatial variation factors computed using 2000 synthetic shear-wave splitting measurements at 100 stations (inset at the upper right corner) from 20 randomly distributed XKS events (upper left corner). Each curve represents $F_v$ values calculated using a different block size, as labeled.

number of blocks, and the *dx* value for each assumed depth of anisotropy.

The compilation and execution of the program were successful on 64-bit Linux and Sun Solaris computers. Given the simplistic nature of the program and the input files, the program should be easily adapted onto other platforms. The CPU time needed for the computation is dependent on the number of XKS measurements, cell size, the area of the study region, and the CPU speed. As a reference, it took about 5 h on a Dell Precision T7400 workstation (which has a clock speed of 2.66 GHz) to produce the results shown in Fig. 1 (See caption of the figure for the numbers of stations and events and other parameters.)

### 3.1. Files required by AnisDep

AnisDep requires a total of eight data or parameter files, which should all be placed under the same sub-directory where the main program resides.

The only file that needs to be created by the user is the one containing the SWS measurements. The columns of the file should be: (1) station longitude, (2) station latitude, (3) event longitude, (4) event latitude, (5) event depth in km, (6) fast polarization direction measured clockwise from the north, (7) splitting time in second, and (8) phase name (*SKS*, *SKKS*, or *PKS*, must be in upper case). Note that the unit for columns 1–4 is decimal degree, and fixed format of the columns is not necessary (see test.dat for an example input file).

The user is not expected to modify the rest of the files, including iasp91.6372, which is a re-sampled (into 1 km vertical interval) IASPEI91 Earth model (Kennett and Engdahl, 1991); iasp91.hed, iasp91.tbl, limits.inc, stdconst, stdparm, and ttlim.inc, which are parameter files in the original IASPEI91 package by Ray Buland and colleagues. Note that if the program is installed on a Sun Solaris computer, the original iasp91.hed and iasp91.tbl files (which are for Linux) should be removed and iasp91.hed.sun and

iasp91.tbl.sun should be renamed as iasp91.hed and iasp91.tbl, respectively, before AnisDep.exe can be executed.

## 3.2. Executing and testing of AnisDep

AnisDep can be executed by typing AnisDep.exe and entering the name of the file containing the SWS measurements. A data set of 2000 synthetic SWS measurements (test.dat) obtained at 100 stations located on a hypothetical network approximately centered at (0°E, 0°N) is provided to test AnisDep. The procedure to generate the synthetic data set is described in Liu and Gao (2011). The synthetic data set corresponds to a depth of anisotropy of 200 km. The GMT program, plot.gmt, can be used to plot the resulting $F_v$ curves (Fig. 1). An application of the procedure to real data is presented in Gao et al. (2010) for the Afar depression and adjacent areas in Ethiopia.

## 3.3. Output files

The program creates eight output files with the name output.dx$a.bc$, where $a.bc$ is $dx$ which is a number between 0.05 and 0.40 and indicates the size (in degree) of the blocks used to create the results. Note that synthetic tests (Liu and Gao, 2011) suggest that reliable results cannot be guaranteed if $dx$ is less than 0.05° or greater than 0.4°. The columns of the output files are (1) depth of anisotropy in km; (2) normalized (by the value at zero depth) variation factor; (3) SD of the normalized variation factor; (4) number of blocks used to produce the variation factor; and (5) block size in degree.

In addition, there are two temporary files, piercing.tmp and cell_mean.tmp which are created and used by AnisDep for temporary data storage.

## 3.4. Potential parameter modifications

The parameters are chosen for typical regional seismic experiments with array dimensions of several hundreds to several thousands km, and for source of anisotropy in the upper mantle (0–400 km). The following parameters can be directly changed by editing the program and recompiling it. $dx$1—the minimum block length. The original value is 0.05°. $dx$2—the maximum block length. The original value is 0.4°. $ddx$—distance between the center of neighboring blocks. The original value is 0.05°. $dmove$—amount of shift of the moving window. $dep$1, $dep$2, and $ddep$—the minimum and maximum depths and the depth increment in km. The original values are 0, 400, and 5 km, respectively.

## 4. Structure of AnisDep and functionalities of the subroutines

The bulk of AnisDep consists of a system of nested do-loops with five-levels. The top level is the loop for the block size ($dx$). The second level is for assumed depth of anisotropy, ranging from $dep$1 to $dep$2 with an interval of $ddep$ km. Those three parameters are set as 0, 400, and 5 km, respectively, and can be modified by the user. At this level, the geographic coordinates of the piercing points at a given depth are calculated, and their spatial extents are determined. Levels 3 and 4 are for the latitude and longitude, respectively, of the blocks. The size of the blocks is defined by the parameter $dx$, and the shifting distance between neighboring blocks is defined by the parameter $dmove$ which is set as 0.05° in the program. Those loops compute the sample mean and SD for the splitting times, and the circular mean and SD for the fast directions for piercing points inside the block. Level 5 is a search of all the splitting measurements for those inside the block.

## 4.1. Functionality of the major subroutines

AnisDep uses 40 subroutines and external functions, including nine that are called in the main program. The subroutine $distazsub$ computes the great-circle-arc distance ($gcarc$) and back-azimuth ($baz$) between the source and receiver, which is used by $tat\_sks$, $tat\_skks$, and $tat\_pks$ to compute the ray parameter ($rayp$) for SKS, SKKS, and PKS, respectively. $rayp$, $gcarc$, and $baz$ are then used to calculate the geographic coordinates of a given ray-path at a given anisotropy depth by subroutine $find\_loc$. The mean and SD of the $\delta t$ measurements inside a block are computed using subroutine $meansig\_sample$, and those of the $\phi$ measurements are computed using $meansig\_circular$2. The main program and some of the subroutines call third-party subroutines written by Ray Buland and colleagues in their IASPEI91 subroutine package. The subroutines for calculating great-circle-arc distance and back-azimuth were written by John Evans and colleagues. These subroutines are openly distributed by the IRIS (Incorporated Research Institutions for Seismology).

### 4.1.1. find_loc

Subroutine $find\_loc$ computes the geographic coordinates of the XKS ray-piercing points at a given depth (the assumed depth of anisotropy). It accomplishes the task by converting the ray-parameter of an XKS ray-path into epicentral distance of a hypothetical local earthquake, which has a focal depth that is the same as the assumed depth of anisotropy, and has the same direct S-wave ray parameter as that of the XKS wave. This is done through the following steps. First, the Earth from the surface to the assumed depth of anisotropy ($D_A$) is divided into 1-km thick layers, and each layer is given a shear-wave velocity by interpolating the IASPEI91 Earth model. Then for the $i$th layer (starting from the surface), the angle of the ray at the top of the layer, $\alpha_i$ (measured from the vertical) is calculated using the Snell's law for a spherically layered Earth (e.g., Eq. (17.12) and Fig. 17.3 in Stacey and Davis, 2008), i.e.,

$$\alpha_i = \arcsin[p*v_i/(R-d_{i-1})] \qquad (7)$$

where $p$ is the ray-parameter (in s/radian), $R$ is the mean radius of the Earth (in km), and $d$ is the depth of the interface (in km). Similarly, the angle of the ray at the bottom of the layer is calculated using

$$\beta_i = \arcsin[p*v_i/(R-d_i)] \qquad (8)$$

We next compute the epicentral distance of the hypothetical local earthquake with a focal depth of $D_A$ by summing up the difference between $\beta$ and $\alpha$, i.e.,

$$\Delta = \sum_{i=1}^{n}(\beta_i - \alpha_i) \qquad (9)$$

where $n$ is the number of layers above $D_A$.

Finally, the geographic coordinates of the ray-piercing points are calculated using simple trigonometric relations based on $\Delta$, the back-azimuth of the XKS ray-path, and the coordinates of the recording station.

### 4.1.2. meansig_circular2

The subroutine $meansig\_circular$2 computes the circular mean and SD of the observed fast directions. It was written based on the formulism provided in the book of Mardia (1972) on directional data analysis.

Because fast directions are directional data with a maximum possible range of 180°, simple arithmetic mean and standard deviations can lead to mean values that are 90° apart and vastly different SD values, depending on whether [−90°, 90°] or [0°, 180°] is used as the $\phi$ range. For instance, the arithmetic mean for 1° and

$179°$ is $90°$ and the sample SD is $125.7°$ when the inappropriate range of $[0°, 180°]$ is used. The corresponding results become $0°$ and $1.414°$ if the $\phi$ range of $[-90°, 90°]$ is used (in this case $179°$ becomes $-1°$). This simple example suggests that the choice of the $\phi$ range must be on the basis of the distribution of the individual $\phi$ measurements. Unfortunately, many previous SWS studies arbitrarily choose one of the two $\phi$ ranges for all the stations, which might lead to erroneous mean and SD values. Moreover, when the number of measurements increases, choosing the correct $\phi$ range becomes less obvious.

Subroutine *meansig_circular*2 calculates circular mean and SD (Mardia, 1972) and consequently produces correct results regardless the $\phi$ range used in the measurements. For $M_i$ measurements of $\phi$, the circular SD is computed using Eq. (3) above, and the circular mean is computed using (Davis, 2002; Mardia and Jupp, 2008)

$$0.5 \tan^{-1}(s/c), \quad s \geq 0, \ c \geq 0 \tag{10}$$

$$0.5[\tan^{-1}(s/c) + 180°], \quad c < 0 \tag{11}$$

$$0.5[\tan^{-1}(s/c) + 360°], \quad s < 0, \ c > 0 \tag{12}$$

where $s = 1/M_i \sum_{j=1}^{M_i} \sin(2\phi_{ij})$ and $c = 1/M_i \sum_{j=1}^{M_i} \cos(2\phi_{ij})$.

## 5. Summary remarks

This report provides detailed descriptions about AnisDep, a FORTRAN program to estimate the depth of the source of anisotropy by measuring spatial coherency of *XKS* shear-wave splitting parameters in a study area. While in principle the procedure can also be used for splitting data sets derived from non-XKS phases such as direct S and ScS, due to possible contamination of source-side anisotropy to the observed splitting parameters, they should be excluded from the data sets to be used in the depth-estimate procedure.

As discussed in greater detail by Liu and Gao (2011), the application of the procedure is limited in areas dominated by simple anisotropy, i.e., a single layer of anisotropy with a horizontal axis of symmetry. Unfortunately, whether a study area is characterized by simple or complex anisotropy is not always trivial, especially when the azimuthal distribution of the *XKS* events is limited and/or there are rapid lateral variations of anisotropic properties. In addition, there should be adequate azimuthal coverage of the events, and the station spacing should be close enough so that the first Fresnel zones of many rays partially or entirely overlap with each other.

## Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.cageo.2012.01.020.

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