ONE DIMENSIONAL TRIANGULAR FIN EXPERIMENT

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(Updated on 26th August 2009, Original version: 3rd November 2000)

1 GENERAL OBJECTIVES

a) To understand a one-dimensional experimental approximation.

b) To understand the art of experimental measurement; in particular, the judicious use of data.

c) To learn a practical method of measuring a convective heat transfer coefficient for a triangular fin using a statistical analysis. Particularly, the MATLAB API (application programmer's interface) and the Levenberg–Marquardt algorithm to solve non-linear least squares problems.

2 INTRODUCTION

While fins are used everyday, it is tempting to believe that the simplified analysis presented in basic textbooks is not a "real world" description or that it is an unrealistic approximation of what might be observed in the laboratory. This experiment is designed to demonstrate that the fin concept is straightforward and accurate.

3 THEORY

Consider a section of the one-dimensional triangular fin as in Figure 1. Note that the root is at a known temperature, T_w, and that the ambient is at a known temperature, T_∞. Heat is lost from the fin by convection, the rate of which is proportional to the heat transfer coefficient, h, of the slanted surface. Presuming conduction with-in the fin to be primarily considered one-dimensional (i.e., the z dimension is effectively infinite and the perimeter can be approximated as 2z since $l \ll z$), an energy balance for a differential slice of the fin takes the form (1)

$$\frac{d}{dx}\left[kA(x)\frac{dT}{dx} \right] = hp(x)(T - T_\infty)\frac{ds}{dx} \qquad (1)$$

where T(x) is the temperature of the fin at a particular dimensional position x (refer "Heat Transfer from Extended Surfaces" section in [1]).

In order to simplify the expression, non-dimensionalize by introducing the following definitions:

$$\theta = T - T_\infty, \ x^* = \frac{x}{L}$$

Taking k and h to be constants, the expression simplifies to

$$\frac{d}{dx^*}\left(x^* \frac{d\theta}{dx^*} \right) = \frac{hl}{k}\left(\frac{L}{l}\right)^2 \left[1 + \left(\frac{l}{L}\right)^2 \right]^{\frac{1}{2}} \theta \qquad (2)$$

Further defining $B = \frac{hl}{k}\left(\frac{L}{l}\right)^2 \left[1 + \left(\frac{l}{L}\right)^2 \right]^{\frac{1}{2}}$, expression (2) simplifies to

$$\frac{d}{dx^*}\left(x^* \frac{d\theta}{dx^*} \right) = B\theta \qquad (3)$$

on expanding:

$$x^* \frac{d^2\theta}{dx^{*2}} + \frac{d\theta}{dx^*} - B\theta = 0$$

which may be recognized as a second-order ordinary differential equation with variable coefficients subject to the following boundary conditions:

$$\theta(x^* = 0) = finite \qquad (4)$$
$$\theta(x^* = 1) = \theta_0 \qquad (5)$$

where $\theta_0 \equiv T_w - T_\infty$ and the solution is

$$\frac{\theta}{\theta_0} = \frac{I_0\left(2\sqrt{Bx^*}\right)}{I_0\left(2\sqrt{B}\right)} \qquad (6)$$

on the right hand side, I_0 is the modified Bessel function of zeroth order.

At this point, if all of the dimensional parameters (h, l, k, L, T_W and T_∞) were known, the axial temperature profile for the fin would be known as a function of x. Of the parameters

listed, h, the convection heat transfer coefficient is the most difficult to determine, especially since its value depends entirely on the flow field characteristics. The present experiment seeks to estimate h from measured quantities.

This method is based upon a statistical analysis of dimensionless position, x^*, and dimensionless temperature, θ data using equation (6). The selected method is referred to as the Method of Non-Linear Least Squares (NLLS) [4,5], a form of non-linear regression. Based upon the assumption that the errors in the experimental measurements follow a Gaussian distribution, the NLLS produces a unique value for the determined constants. That is, the magnitudes of the constants that are determined give the "most probable" form of the given equation that fits the data. The NLLS is based upon the differences between the independent values of collected experimental data and the expected values as provided by the theoretical result.

For convenience, let $\theta_i = \theta / \theta_0$, then from the theory at each position x_i,

$$\theta_i = DI_0\left(2\sqrt{Bx_i^*}\right) \tag{7}$$

where $D = 1/I_0\left(2\sqrt{B}\right)$ is a constant to be determined so as to fit that experimental data.

For simplicity, let $$I_i = I_0\left(2\sqrt{Bx_i^*}\right) \tag{8}$$

In this case, the residual for each data point is
$$\rho_i = \theta_i - DI_i \tag{9}$$

Note that the dimensionless temperature θ_i, and the dimensionless position x_i^* (or the modified Bessel function at a specific position, I_i) are the input data. Then the sum of the residuals squared is $$S = \sum \rho_i^2 \tag{10}$$

The value of h that would give the least value of residuals squared sum S is found by writing a MATLAB program using the Levenberg–Marquardt algorithm.

Refer to Appendix A at the end for MATLAB API (application programmer's interface) on the Levenberg–Marquardt algorithm.

4   EXPERIMENTAL SETUP

The configuration and schematic of the experimental setup (1-D triangular fin) is presented in Figs. 1 and 2 respectively. The sides and the bottom are all well insulated. This setup represents a physical approximation to the upper half of a symmetric, infinitely long fin. Figure 3 presents the actual experimental fin information.

The temperature indicators are thermocouples embedded within the stainless steel fin as indicated in Figure 4.

5   DATA ACQUISITION PROCEDURE

a.   Familiarize yourself completely with the experimental setup. Do <u>NOT,</u> under any circumstances, touch the stainless steel surfaces because the setup will be turned on before class. Figure 3 is included so that you may view the setup without taking it apart and subjecting yourself to harm.

b.   Confirm that the temperature profile is at steady state (not a function of time). Do not adjust the heater temperature setting. You should have data to prove that steady state exists.

c.   Take the data as per the included data sheet, including T_∞. Repeat the data collection process to confirm the steadiness of the temperatures.

6   DATA REDUCTION PROCEDURE

a.   First plot T (or θ) versus x in order to check the data for the appropriate shape as well as to check its one-dimensional character. Note that T_w is the arithmetic average of the temperature readings at the fin root.

b.   If the shape of the temperature variation is appropriate, enter the data sets (T_i and x_i) and T_∞ into the MATLAB program to solve for 'h' using Levenberg–Marquardt algorithm (non-linear least squares method).

c.   Run the MATLAB program and read the convection coefficient, h.

7   POINTS OF INTEREST

In your technical memorandum you should include a discussion of the following:

1)   The physical significance of the boundary conditions listed by Equations (4) and (5).

2)   Explain clearly how insulation on the two sides simulates a fin which is infinitely long in the z direction.

3)   Why is it important for the fin to be at steady state?

4)   Plot the resulting model (T_i from θ_i using equation (7) with h value from MATLAB program and calculated values of D and B using h) and the measured steady state temperature versus x. Discuss the accuracy of the model.

5)   As an appendix, consider a differential element of the length dx of the experimental fin and obtain the governing equation for the fin. Apply the boundary conditions and hence obtain Equation 6. State the assumptions you made in the process of obtaining Equation 6. Are they all justified for the experimental setup used? Explain. (refer [1])

REFERENCES

1. Incropera, F.P., Fundamentals of Heat and Mass Transfer, John Wiley & Sons, Inc., New Jersey, 2007, pp. 137-162.

2. Chapman, A.J., Heat Transfer, Fourth ed., Macmillan Publishing Company, New York, Collier Macmillan Publishers, London, 1984, pp. 67-89.

3. Hildebrand, F.B., Advanced Calculus for Applications, second ed., Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1976, pp. 141-153.

4. Bain, L.J., and M.Engelhardt, Introduction to Probability and Mathematical Statistics, PWS Publishers, 1987, pp. 300-307.

5. Lipson, C., and N.J. Sheth, Statistical Design of Engineering Experiments, McGraw-Hill Book Company, 1973, pp. 372-387.

6. Levenberg–Marquardt algorithm, www.wikipedia.org

NOMENCLATURE

h = surface convection coefficient for a triangular fin (W/m² ^{0}C)

k = thermal conductivity (W/m ^{0}C)

l = one half fin thickness at the root (m)

L = fin length (m)

s = slant length (m)

T = fin temperature (^{0}C)

T_w = fin root temperature (^{0}C)

T_∞ = ambient temperature (^{0}C)

x = dimensional axial coordinate along fin length (m)

x^* = x/L, dimensionless axial coordinate

y = coordinate along fin height

z = coordinate along fin width

A(x) = cross-sectional area parallel to the wall and enclosed by the perimeter

    (i.e. 2zy=2zx$l$/L)

p(x) = perimeter of the cross sectional area of the triangular fin (i.e. 2(z+2y) = 2z, as

    z>>2y from a thermodynamic point of view)

ds = $((dx)^2 + (dy)^2)^{1/2}$

$\theta = (T - T_\infty)$

$\theta_0 = T_w - T_\infty$
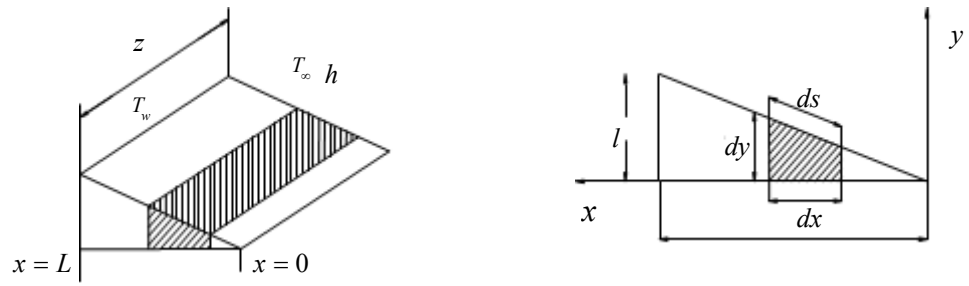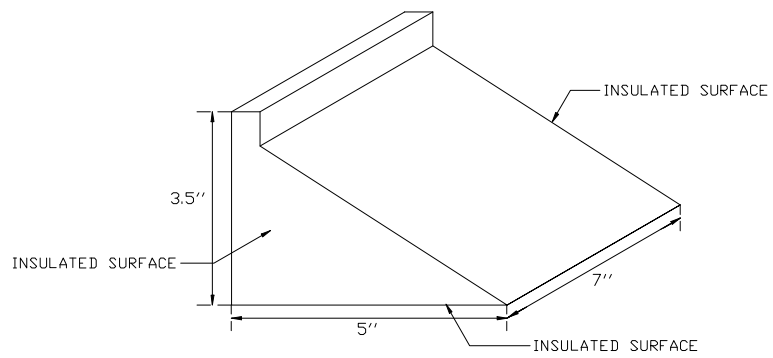
One Dimensional Triangular Fin Experimental Data Sheet

Date: $T_w =$

Name: $T_\infty =$

TC#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Depth from Fin Bottom (in)	0	1.2	0	0.1	0.5	0.8	0.3	0	1.0	0.4	0.6	0.2	1.5	0	0.9	0.4
Distance from root (in)	0	1.25	2.5	4.5	0	1.25	2.5	3.75	0	1.25	2.5	3.75	0	1.25	2.5	3.75

Temperature (F)

TC# time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Figure 1 Configuration of triangular fin

Figure 2. Schematic of 1-D triangular fin

protective cover box
(heater inside)

heater on back
surface

control panel used to
turn on the experiment

thermocouple selector
switch allows from
only one thermocouple
to be displayed

cable between selector
device and display device

temperature
display device

232.5

Figure3. Schematic of experimental setup

ROOT

a)

.13

.9   . 14

.   . 10   . 15
.5   . 6   . 11   .16
.   .   . 7   .12
1   2   3   8   . 4

insulation

ROOT

TIP

.3   .14   .15   .16
9   .10   .11   .12
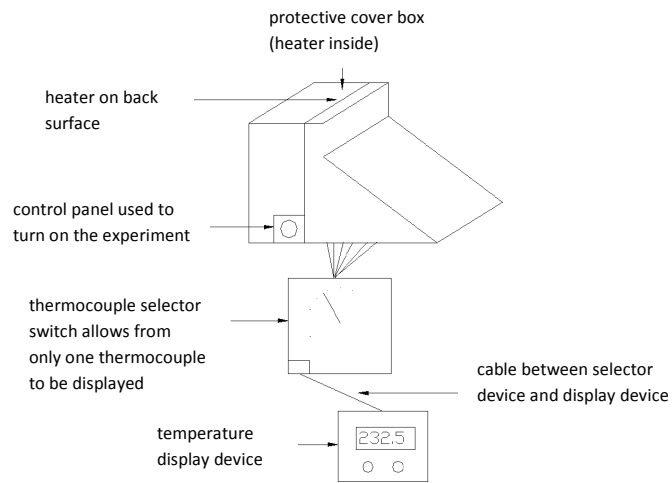5   . 6   . 7   . 8
1   . 2   . 3   . 4
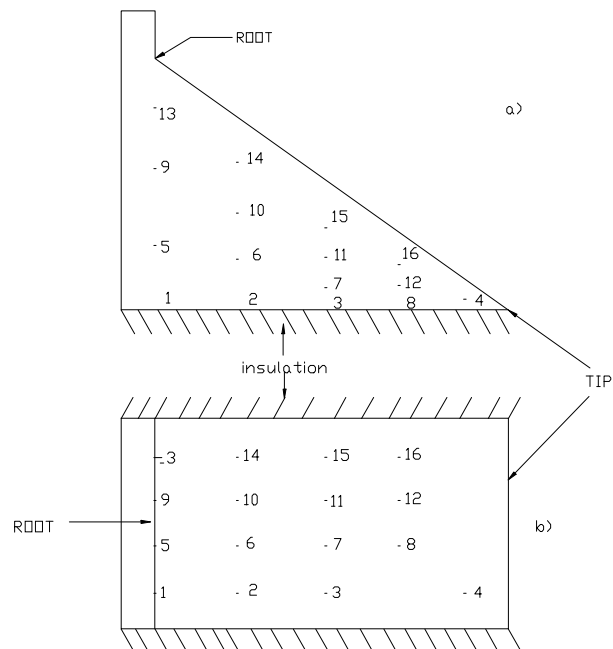
b)

Figure 4. Thermocouple position
        a) side view
        b) top view

APPENDIX A

MATLAB API (application programmer's interface) for the Levenberg–Marquardt algorithm to solve non-linear least squares problems.

Example: Rosenbrock function - in mathematical optimization, the Rosenbrock function is a non-convex function used as a test problem for optimization algorithms. The global minimum is inside a long, narrow, parabolic shaped flat valley. To find the valley is trivial, however to converge to the global minimum is difficult. The Rosenbrock function is defined as

$$f(x,y) = (1-x)^2 + 100(y-x^2)^2$$

To test if Levenberg–Marquardt algorithm is able to find the global minimum (1,1): create a new file in MATLAB with the following commands (make sure the new MATLAB file and LMFsolve.m, Levenberg–Marquardt algorithm are in the same working directory). In this case m=[x;y], m(1)=x and m(2)=y.

```
rosenbrock= @(m) [1-m(1); 10*(m(2)-m(1)^2)];
[m,residuals_squares_sum,iterations]=LMFsolve(rosenbrock,[-1.2,3],'Display',1,'MaxIter',15000)
```

"1-m(1)" is the first function and "10*(m(2)-m(1)^2)" is the second function

with Levenberg–Marquardt algorithm we are trying to find the minimum of the squares-sum of these two functions.

[-1.2,3] is the initial guess value for m

"'Display',1,'" is for displaying iteration information after every iteration

"'MaxIter',15000" is to set the maximum number of iterations to 15000

Running the program will return "m = [1; 1]" i.e, m(1) = x = 1 and m(2) = y = 1 (in approximately 4000 iterations) which is the global minimum for the Rosenbrock function.

Fin Experiment: Develop a new MATLAB program for this experiment with the help of the example as shown above.

Hint: The important commands are-

```
residuals= @(h) [ρ_a;ρ_b;ρ_c;ρ_d;ρ_e];
[h,residuals_squares_sum,iterations]=LMFsolve(residuals,[100],'Display',1,'MaxIter',15000)
```

where ρ is the error (equation 9) at different x values as a function of h
"100" is the initial guess value for h